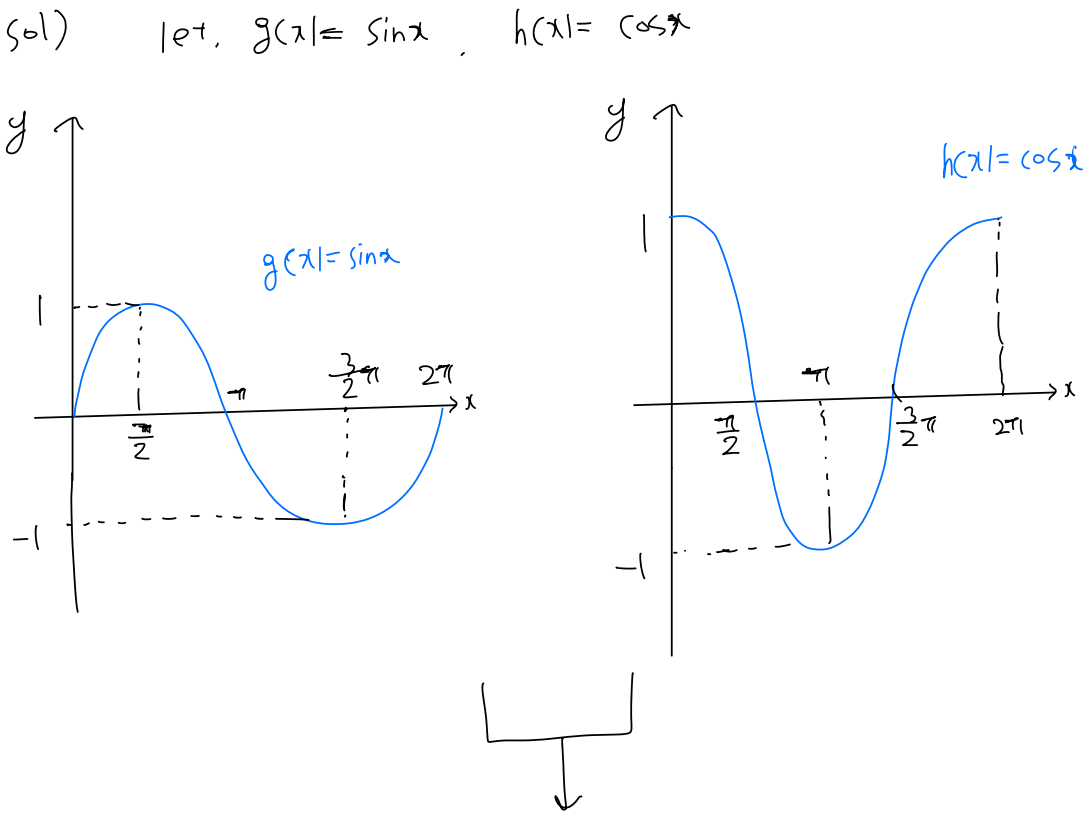
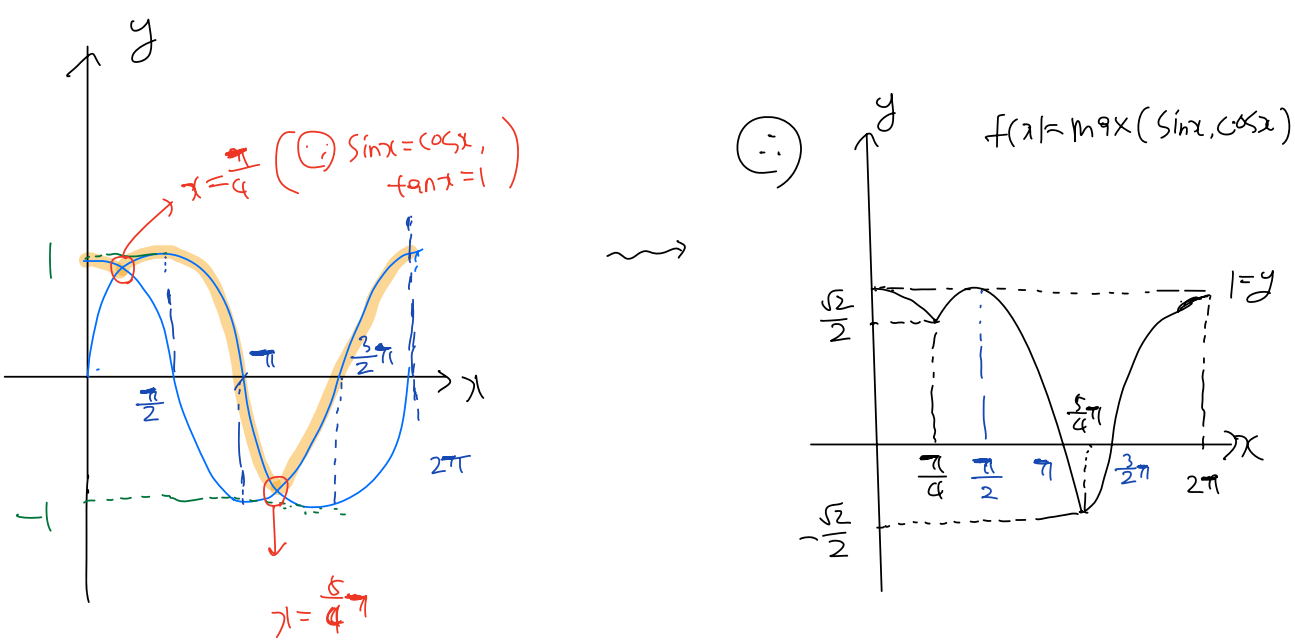
1. max{a,b,c...}는 a,b,c,... 중에서 가장 큰 값을 의미한다고 하자. 이 때, 다음 함수를 구간 $[0,2\pi]$에서 그려보시오. **[20pts]**

$f(x)=\max(\sin x,\cos x)$





2. 만일 $x > 0$이고 $x \neq 1$에서 다음 식이 성립함을 보이시오. [20pts]

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_5 x} = \frac{1}{\log_{30} x}$$

Sol) $\log_2 x = \frac{\log_{30} x}{\log_{30} 2}$, $\log_3 x = \frac{\log_{30} x}{\log_{30} 3}$, $\log_5 x = \frac{\log_{30} x}{\log_{30} 5}$

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_5 x}$$

$$= \frac{(\log_3 x \cdot \log_5 x) + (\log_2 x \cdot \log_5 x) + (\log_2 x \cdot \log_3 x)}{(\log_2 x \cdot \log_3 x \cdot \log_5 x)}$$

$$= \frac{(\log_{30} x)^2 \left(\frac{1}{\log_{30} 3} \cdot \frac{1}{\log_{30} 5}\right) + (\log_{30} x)^2 \left(\frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 5}\right) + (\log_{30} x)^2 \left(\frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 3}\right)}{(\log_{30} x)^3 \cdot \left(\frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 5} \cdot \frac{1}{\log_{30} 3}\right)}$$

$$= \frac{\frac{1}{\log_{30} 5}\left(\frac{1}{\log_{30} 3} + \frac{1}{\log_{30} 2}\right) + \frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 3}}{(\log_{30} x)\left(\frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 5} \cdot \frac{1}{\log_{30} 3}\right)}$$

$$= \frac{\frac{1}{\log_{30} 5}\left(\frac{\log_{30} 3 + \log_{30} 2}{\log_{30} 3 \cdot \log_{30} 2}\right) + \frac{1}{\log_{30} 2 \cdot \log_{30} 3}}{(\log_{30} x)\left(\frac{1}{\log_{30} 2} \cdot \frac{1}{\log_{30} 5} \cdot \frac{1}{\log_{30} 3}\right)}$$

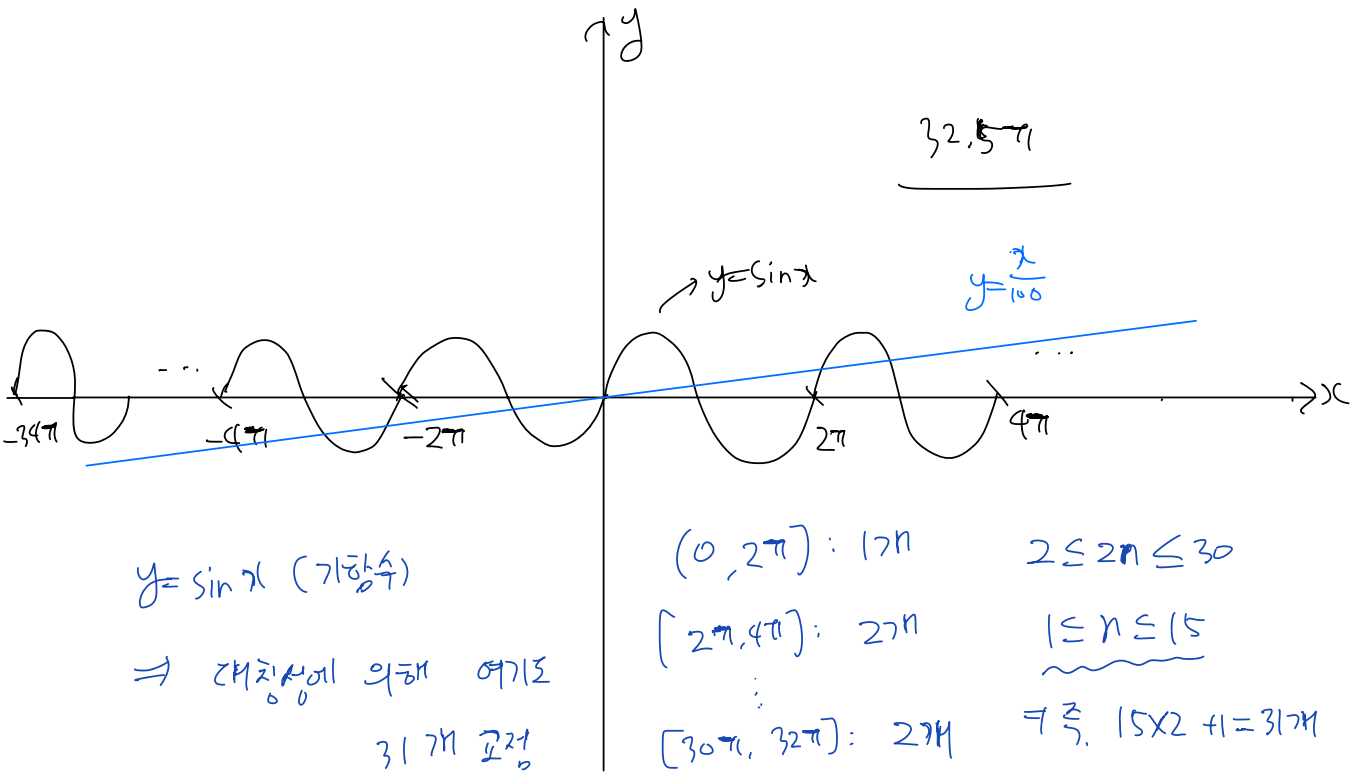
$$= \frac{\log_{30}6 + \log_{30}5}{\log_{30}2 \cdot \log_{30}3 \cdot \log_{30}5} \quad (\log_{30}6 + \log_{30}5 = \log_{30}30 = 1)$$

$$= \frac{1}{(\log_{30}x)\left(\frac{1}{\log_{30}2} \cdot \frac{1}{\log_{30}5} \cdot \frac{1}{\log_{30}3}\right)}$$

$$= \frac{1}{(\log_{30}x)\left(\frac{1}{\log_{30}2} \cdot \frac{1}{\log_{30}5} \cdot \frac{1}{\log_{30}3}\right)(\log_{30}2 \cdot \log_{30}3 \cdot \log_{30}5)}$$

$$= \frac{1}{\log_{30}x}$$

3. 방정식 $\sin x = \frac{x}{100}$ 에 대하여 이 방정식의 해는 몇 개인지 설명하시오. **[20pts]**



$y = \sin x$ (기함수)

⇒ 대칭성에 의해 여기도 31개 고정

$(0, 2\pi]$: 1개

$[2\pi, 4\pi]$: 2개

⋮

$[30\pi, 32\pi]$: 2개

$2 \le 2n \le 30$

$1 \le n \le 15$

⇒ 즉, 15×2 +1 = 31개

☺ $31 + 31 + 1 = 63$개

원점

4. 다음 물음에 답하시오. **[40 pts]**

a) $f(x)=\ln(x+\sqrt{x^2+1})$ 은 기함수인가? 우함수인가? 설명하시오. **[20 pts]**

b) 함수 f의 역함수 형태를 구해보시오. (단, 과정을 상세히 서술하시오.) **[20 pts]**

a) $f(x)=\ln(x+\sqrt{x^2+1})$

$f(-x)=\ln(-x+\sqrt{x^2+1})$

let, $f(x)=-f(-x)$ 라고 가정하자

$\Rightarrow \ln(x+\sqrt{x^2+1})=-\ln(-x+\sqrt{x^2+1})$ 임을 보이자.

i) $\ln(x+\sqrt{x^2+1})$

$=\ln\left\{\dfrac{(x+\sqrt{x^2+1})(x-\sqrt{x^2+1})}{(x-\sqrt{x^2+1})}\right\}$

$=\ln\left(\dfrac{-1}{x-\sqrt{x^2+1}}\right)$

$=\ln\left(\dfrac{1}{-x+\sqrt{x^2+1}}\right)$

ii) $-\ln(-x+\sqrt{x^2+1})$

$=\ln\left(\dfrac{1}{-x+\sqrt{x^2+1}}\right)$ 이므로

i)=ii) 이다

$\therefore f(x)=-f(-x)$

f는 기함수

b) $y = \ln(x + \sqrt{x^2+1})$

$\Leftrightarrow e^y = x + \sqrt{x^2+1}$

$\Rightarrow (e^y - x) = \sqrt{x^2+1}$

$\Rightarrow e^{2y} - 2e^y \cdot x + x^2 = x^2+1$

$\Rightarrow e^{2y} - 2xe^y = 1$

$2xe^y = e^{2y} - 1$

$\Rightarrow x = \frac{e^{2y}-1}{2e^y} = \frac{e^y - e^{-y}}{2} = \sinh y$

$\therefore f^{-1}(x) = \sinh y = \frac{e^y - e^{-y}}{2}$