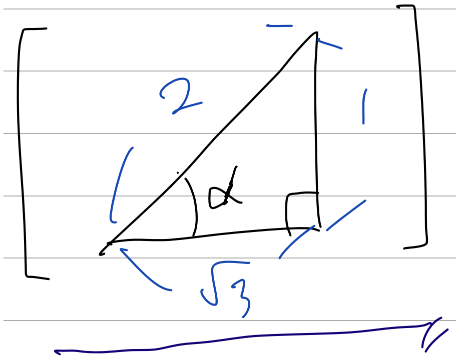
유형 1.

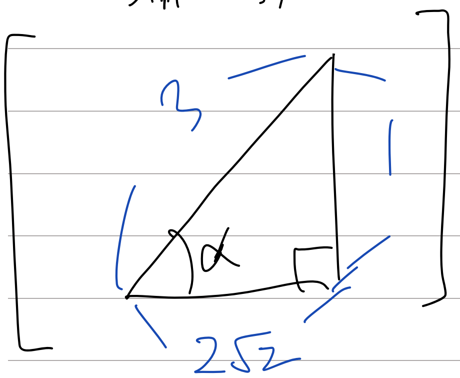
1. $\sin^{-1}\left(\frac{1}{2}\right) \triangleq \alpha \iff \sin\alpha = \frac{1}{2} \quad \left(-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}\right)$



$\sin\alpha = \frac{1}{2}$. $\alpha = \frac{\pi}{6}$ //

2. $\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

$\sin^{-1}\left(\frac{1}{3}\right) \triangleq \alpha \iff \sin\alpha = \frac{1}{3} \quad \left(-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}\right)$



$\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

$= \tan\alpha = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$ //

3. $y = \sin^{-1}(3x+1) \iff \sin y = 3x+1 \quad \left(-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right)$

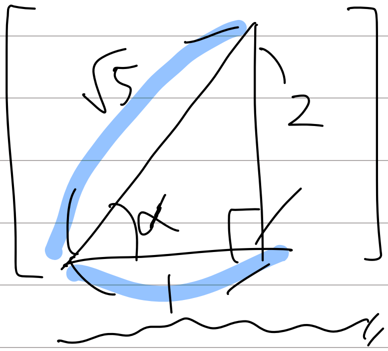
$-1 \le \sin y \le 1$

$-1 \le 3x+1 \le 1 \rightarrow -2 \le 3x \le 0$

$-\frac{2}{3} \le x \le 0$ //

4. $\sec(\tan^{-1}(2))$

$\tan^{-1}(2) \stackrel{\text{은}}{=} \alpha \Leftrightarrow \tan\alpha = 2 \quad \left(-\frac{\pi}{2} < \alpha < \frac{\pi}{2}\right)$



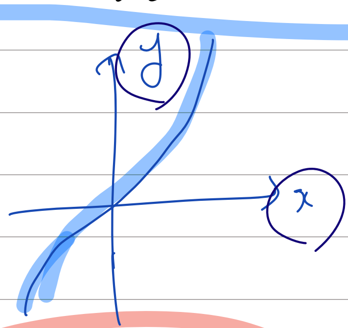
$\sec(\tan^{-1}(2))$

$= \sec\alpha$

$= \dfrac{1}{\cos\alpha} = \sqrt{5}$

유형2. ① 주치범위
② 쌍곡선함수의 정의 + 근의공식활용 (이차방정식)

1. $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$



sol) $y = \sinh^{-1}x \Leftrightarrow \sinh y = x \quad (y \in \mathbb{R},\ x \in \mathbb{R})$

$\sinh y = \dfrac{e^y - e^{-y}}{2} = x$

$\Rightarrow e^y - e^{-y} = 2x$

$\Rightarrow e^{2y} - 1 = 2x e^y$

$\Rightarrow e^{2y} - 2x e^y - 1 = 0$

$(e^y = t \quad t > 0)$

$t^2 - 2xt - 1 = 0$

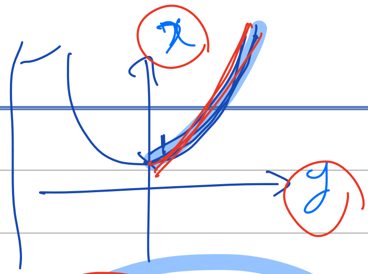
$t = x \pm \sqrt{x^2+1}$

$= x + \sqrt{x^2+1} = e^y$

($\because$ $t>0$)

$\boxed{y = \ln\left(x + \sqrt{x^2+1}\right)}$

2. $\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right)$



$y=\cosh^{-1}x \iff \cosh y = x \quad (y \ge 0,\ x \ge 1)$

$$\cosh y = \frac{e^y+e^{-y}}{2} = x$$

$$e^y+e^{-y} = 2x$$

$$e^{2y}+1 = 2x\cdot e^y$$

$$e^{2y}-2xe^y+1=0$$

$(e^y = t,\ t \ge 1)$

$\Rightarrow t^2-2xt+1=0$

$\Rightarrow t = x \pm \sqrt{x^2-1}$

$$\boxed{\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right) = x^2-(x^2-1) = 1}$$

$\Rightarrow t = x+\sqrt{x^2-1} = e^y$

$\Rightarrow y = \ln\left(x+\sqrt{x^2-1}\right)$

3. $y=\tanh^{-1}x \iff \tanh y = x \quad (y\in\mathbb{R},\ -1<x<1)$

$$\tanh y = \frac{e^y-e^{-y}}{e^y+e^{-y}} = x$$

$$e^y-e^{-y} = x\left(e^y+e^{-y}\right)$$

$$e^{2y}-1 = x\left(e^{2y}+1\right)$$

$$(1-x)e^{2y} = x+1$$

$$e^{2y} = \frac{x+1}{1-x}$$

$$e^y = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}}$$

$\Rightarrow y = \ln\left(\frac{x+1}{1-x}\right)^{\boxed{\frac{1}{2}}}$

$= \frac{1}{2}\ln\left(\frac{x+1}{1-x}\right)$

4. $\operatorname{sech}^{-1} x = y \Leftrightarrow \operatorname{sech} y = x \quad (y \ge 0,\ 0 < x \le 1)$

$\operatorname{sech} y = \dfrac{2}{e^{y}+e^{-y}} = x$

$(e^{y}+e^{-y})x = 2$

$(e^{2y}+1)x = 2e^{y}$

$x \cdot e^{2y} - 2e^{y} + x = 0$

$e^{y} = \dfrac{1 \pm \sqrt{1-x^2}}{x}$

$e^{y} = \boxed{\dfrac{1+\sqrt{1-x^2}}{x}}$

$(x \to 0,\ \operatorname{sech}^{-1} x \to \infty)$

$y = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1-x^2}}{x}\right)$

5. $y = \operatorname{csch}^{-1} x \Leftrightarrow \operatorname{csch} y = x \quad (y \ne 0,\ x \ne 0)$

$\operatorname{csch} y = \dfrac{2}{e^{y}-e^{-y}} = x$

$2 = (e^{y}-e^{-y})x$

$2e^{y} = (e^{2y}-1)x$

$x \cdot e^{2y} - 2e^{y} - x = 0$

$e^{y} = \dfrac{1 \pm \sqrt{1+x^2}}{x}$

$e^{y} > 0$

$e^{y} = \dfrac{1+\sqrt{1+x^2}}{x}$

$y = \ln\left(\dfrac{1+\sqrt{1+x^2}}{x}\right)$

6. $y = \coth^{-1} x \iff \coth y = x \quad (y \neq 0,\ |x| > 1)$

$$\coth y = \frac{e^y + e^{-y}}{e^y - e^{-y}} = x$$

$$e^y + e^{-y} = x(e^y - e^{-y})$$

$$e^{2y} + 1 = x(e^{2y} - 1)$$

$$e^{2y}(1-x) = -x-1$$

$$e^{2y}(x-1) = x+1$$

$$e^{2y} = \frac{x+1}{x-1}$$

$$e^y = \left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}$$

$$y = \ln\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

유형3

1. $\lim_{x \to -2} \dfrac{\boxed{3x^2+ax+a+3}}{\boxed{x^2+x-2}}$ $\quad 3x^2+ax+a+3 \to 0, \quad x^2+x-2 \to 0$

$= \lim_{x \to -2} \dfrac{\boxed{3x^2+ax+a+3}}{x^2+x-2}$ $\Rightarrow 12-2a+a+3=0$

$-a+15=0, \quad a=15$

$\lim_{x \to -2} \dfrac{3x^2+15x+18}{x^2+x-2}$

$= \lim_{x \to -2} \dfrac{3(x^2+5x+6)}{x^2+x-2}$

$= \lim_{x \to -2} \dfrac{3\cancel{(x+2)}(x+3)}{(x-1)\cancel{(x+2)}}$ $\quad \dfrac{0}{0}$

$= \lim_{x \to -2} \dfrac{3(x+3)}{(x-1)} = \dfrac{3}{-3} = -1$

2. $\lim_{x\to0} \boxed{\dfrac{f(x)}{x^2}} = 5$

$$\begin{cases} \lim_{x\to0} f(x) \\ \lim_{x\to0} \dfrac{f(x)}{x} \end{cases}$$

① $\lim_{x\to0} f(x)$

$$\frac{f(x)}{x^2} \cdot x^2 = f(x)$$

$$\lim_{x\to0} \frac{f(x)}{x^2} = 5 \quad \& \quad \lim_{x\to0} x^2 = 0$$

$$\lim_{x\to0} \left(\frac{f(x)}{x^2} \cdot x^2 \right) = \lim_{x\to0} f(x)$$

$$= \lim_{x\to0} \left(\frac{f(x)}{x^2} \right) \cdot \lim_{x\to0} x^2$$

$$= 5 \cdot 0 = 0$$

② $\lim_{x\to0}\boxed{\dfrac{f(x)}{x}}$

$$\frac{f(x)}{x^2}\cdot x=\boxed{\frac{f(x)}{x}}\quad \begin{cases}\lim_{x\to0}\dfrac{f(x)}{x^2}=5\\ \lim_{x\to0}x=0\end{cases}$$

$$\lim_{x\to0}\frac{f(x)}{x}=\lim_{x\to0}\left(\frac{f(x)}{x^2}\cdot x\right)$$

$$=\lim_{x\to0}\frac{f(x)}{x^2}\cdot\lim_{x\to0}x$$

$$=5\cdot0=0$$

3. $\lim_{x\to2}\dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$ (유리화!)

$$=\lim_{x\to2}\frac{(\sqrt{6-x}-2)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}{(\sqrt{3-x}-1)(\sqrt{3-x}+1)(\sqrt{6-x}+2)}$$

$$=\lim_{x\to2}\frac{\cancel{(2-x)}(\sqrt{3-x}+1)}{\cancel{(2-x)}(\sqrt{6-x}+2)}$$

$$= \lim_{x \to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \frac{2}{4} = \frac{1}{2}$$

4. $$\lim_{x \to 0} \frac{\cot 2x}{\csc x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \Rightarrow \lim_{x \to 0} \frac{\sin bx}{ax} = \frac{b}{a}$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1 \Rightarrow \lim_{x \to 0} \frac{\tan bx}{ax} = \frac{b}{a}$$

$$\Downarrow$$

$$\lim_{x \to 0} \frac{\sin bx}{\tan ax} = \frac{b}{a}$$

$$\lim_{x \to 0} \frac{\cot 2x}{\csc x}$$

$$= \lim_{x \to 0} \frac{\sin x}{\tan 2x}$$

$$= \lim_{x \to 0} \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{2x}{\tan 2x}}_{\to 1} \cdot \frac{x}{2x}$$

$$= \lim_{x\to 0} \frac{\cancel{x}}{2\cancel{x}} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$$

5. $$\boxed{\lim_{x\to 0^+} (1+\sin 4x)^{\frac{\cot x}{8}}}$$ 1^∞

$$\frac{\cos x}{8\sin x} \to \frac{1}{0} = \infty$$

$$\lim_{x\to a} \frac{g(x)}{f(x)} \quad \frac{\infty}{\infty} \quad \frac{0}{0} \quad \lim_{x\to a} \frac{g'(x)}{f'(x)}$$

$$1^\infty \to e^{\ln} = 로피탈$$

$$\lim_{x\to 0^+} (1+\sin 4x)^{\frac{\cot x}{8}}$$

$$= \lim_{x\to 0^+} e^{\boxed{\frac{\cot x}{8}} \ln(1+\sin 4x)}$$

$$= \lim_{x\to 0^+} e^{\frac{\cot x \ln(1+\sin 4x)}{8}}$$

$$= \lim_{x\to 0^+} e^{\frac{\cancel{\cos x} \ln(1+\sin 4x)}{8\sin x}}$$

$$\lim_{x\to 0^+} e^{\frac{\ln(1+\sin 4x)}{8\sin x}}$$

$$\frac{0}{0} = \lim_{x\to 0^+} e^{\frac{4\cos 4x}{8\cos x\,(1+\sin 4x)}}$$

$$(\sin x)' = \cos x$$

$$\{\ln(1+\sin 4x)\}' = \frac{4\cos 4x}{1+\sin 4x}$$

$$\ln|f(x)| = \frac{f'(x)}{f(x)} = \frac{\cos 4x \cdot 4}{1+\sin 4x}$$

$$= \lim_{x\to 0^+} e^{\frac{4}{8(1+\sin 4x)}}$$

$$= e^{\frac{1}{2}} = \sqrt{e}$$

6. $\lim_{x\to0^+} \dfrac{x^{\frac{1}{6}}\sin x}{(e^x-1)^{\frac{8}{7}}}$

$\left(\lim_{x\to0} \dfrac{e^x-1}{x} = 1 \right.$ $(e^x-1\to0,\ x\to0)$ & $\left. \lim_{x\to0} \dfrac{\sin x}{x} = 1 \right)$

$\sin x$ / $\tan x$ / e^x-1

$\lim_{x\to0} \dfrac{e^x}{1} = 1$

$\lim_{x\to0^+} \dfrac{x^{\frac{1}{6}}\sin x}{(e^x-1)^{\frac{8}{7}}}$

$= \lim_{x\to0^+} \dfrac{x^{\frac{8}{7}}}{(e^x-1)^{\frac{8}{7}}} \cdot \dfrac{\sin x}{x} \cdot \dfrac{x^{\frac{1}{6}}\cdot x}{x^{\frac{8}{7}}}$

$\left(\dfrac{x}{e^x-1}\right)^{\frac{8}{7}} = 1$

$= \lim_{x\to0^+} \dfrac{x^{\frac{7}{6}}}{x^{\frac{8}{7}}} = \lim_{x\to0^+} x^{\frac{1}{42}} = 0$

8. $\lim_{x\to\infty} (a^x + b^x)^{\frac{1}{x}}$

$= \lim_{x\to\infty} e^{\ln (a^x+b^x)^{\frac{1}{x}}}$

$= \lim_{x\to\infty} e^{\frac{\ln(a^x+b^x)}{x}}$

case1) $a=b=1$

$\lim_{x\to\infty} e^{\frac{\ln(1+1)}{x}} = \lim_{x\to\infty} e^{\frac{\boxed{\ln 2}}{x}} = e^0 = 1$ //

case2) $a \geq b$

$\lim_{x\to\infty} e^{\frac{\ln(a^x+b^x)}{x}}$

$(a^x)' = a^x \cdot \ln a$

$\ln(a^x+b^x)' = \frac{a^x \ln a + b^x \ln b}{a^x+b^x}$

$\frac{\infty}{\infty}$

$= \lim_{x\to\infty} e^{\frac{a^x \ln a + b^x \ln b}{a^x+b^x}}$

$= \lim_{x\to\infty} e^{\frac{\cancel{a^x}\left(\ln a + \cancel{\left(\frac{b}{a}\right)^x}\ln b\right)}{\cancel{a^x}\left(1+\cancel{\left(\frac{b}{a}\right)^x}\right)}}$ ($\left(\frac{b}{a}\right)^x \to 0$)

$= \lim_{x\to\infty} e^{\ln a} = \lim_{x\to\infty} a^{\ln e = 1} = a$ //

Case 3) $a<b$

$$\lim_{x\to\infty} e^{\frac{\ln(a^x+b^x)}{x}}$$

$$\overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} e^{\frac{a^x\ln a+b^x\ln b}{a^x+b^x}}$$

$$= \lim_{x\to\infty} e^{\frac{\cancel{b^x}\left\{\ln b+\left(\frac{a}{b}\right)^x \ln a\right\}}{\cancel{b^x}\left\{\left(\frac{a}{b}\right)^x+1\right\}}}, \quad \left(\frac{a}{b}\right)^x \to 0$$

$$= \lim_{x\to\infty} e^{\ln b} = b$$

9.

$$\lim_{x\to 1}\left(\frac{ax}{x-1}+\frac{b}{\ln x}\right)=1$$

$$\lim_{x\to 1}\frac{ax\ln x+b(x-1)}{(x-1)\ln x}, \quad (x-1)\ln x \to 0$$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 1}\frac{a\ln x+a+b}{\ln x+\frac{x-1}{x}}$$

$a + b \to 0$, $\ln x + \frac{x-1}{x} \to 0$

$\boxed{a+b=0}$

$a=-b$

$$\lim_{x\to1} \frac{a\ln x}{\ln x + \frac{x-1}{x}} \qquad = 1 - \frac{1}{x}$$

$\frac{0}{0}$

$$= \lim_{x\to1} \frac{\frac{a}{x}}{\frac{1}{x} + \frac{1}{x^2}} = 1 = \frac{a}{2}$$

$$\boxed{\therefore\ a=2,\quad b=-2}$$

10. $$\lim_{x\to\infty} \frac{x - \sin x}{x + \sin x} \qquad \lim_{x\to0} \frac{\sin x}{x} = 1$$

$$= \lim_{x\to\infty} \frac{1 - \frac{\sin x}{x}}{1 + \frac{\sin x}{x}} \qquad \lim_{x\to\infty} \frac{\sin x}{x \to \infty} = 0$$

$$= 1$$

11. $$\lim_{x\to\infty} \frac{\frac{1}{x^2} - 2\tan^{-1}\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

$\frac{1}{x} = t$

$\left(\frac{1}{x} = t \Rightarrow x \to \infty,\ t \to 0 \right)$

$$\lim_{t\to 0} \frac{t^2 - 2\tan^{-1}(t)}{t} \quad \to 0$$

$\frac{0}{0}$

$$\boxed{[\tan^{-1}(t)]' = \frac{1}{1+t^2}}$$

외움

$$= \lim_{t\to 0} \frac{2t - \frac{2}{1+t^2}}{1}$$

$$= \lim_{t\to 0} \frac{0-2}{1} = -2$$

12. $$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}}$$

$\sin^2 x + \cos^2 x = 1$

$$\lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} \quad (1+\cos x) \to 2$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \frac{1}{2}$$

$$= \frac{1}{2}$$

(1) $$\lim_{x\to 0} \frac{\overset{1+1}{2} - \cos 3x - \cos 4x}{x} = 0$$

$$= \lim_{x\to 0} \frac{(1-\cos 3x) + (1-\cos 4x)}{x}$$

$$= \lim_{x\to 0} \left\{ \boxed{\frac{1-\cos 3x}{x}} + \boxed{\frac{1-\cos 4x}{x}} \right\}$$

$$= \lim_{x\to 0} \left\{ \frac{\overbrace{(1-\cos 3x)(1+\cos 3x)}^{\sin^2 3x}}{x(1+\cos 3x)} + \frac{\overbrace{(1-\cos 4x)(1+\cos 4x)}^{\sin^2 4x}}{x(1+\cos 4x)} \right\}$$

$$= \lim_{x\to 0}\left\{ \underbrace{\frac{\sin^2 3x}{x(1+\cos 3x)}}_{0} + \underbrace{\frac{\sin^2 4x}{x(1+\cos 4x)}}_{0} \right\}$$

① $$\lim_{x\to 0} \frac{\sin^2 3x}{x(1+\cos 3x)}$$

$$= \lim_{x\to 0} \frac{\sin 3x \cdot \sin 3x}{x\,\boxed{(1+\cos 3x)}} \quad =2$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\sin 3x}{x}\rightarrow 3 \cdot \sin 3x \rightarrow 0 \quad = 0$$

② $$\lim_{x\to 0} \frac{\sin^2 4x}{x(1+\cos 4x)}$$

$$= \lim_{x\to 0} \frac{\sin 4x \cdot \sin 4x}{x\,(1+\cos 4x)} \rightarrow 2$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\sin 4x}{x}\rightarrow 4 \cdot \sin 4x \rightarrow 0 \quad = 0$$

(2) $\lim_{x \to 2} \dfrac{\cos\left(\frac{\pi}{x}\right)}{x-2}$ $\to 0$, $\to 0$

$\sin\left(\frac{\pi}{2} - t\right) = \cos t$

$\cos\left(\frac{\pi}{2} - t\right) = \sin t$

$\left(\frac{\pi}{2} - \frac{\pi}{x}\right)$ 을 t

$\frac{\pi}{x} = \frac{\pi}{2} - t = \frac{\pi - 2t}{2}$

$\exists\ x(\pi - 2t) = 2\pi$. $\boxed{x = \dfrac{2\pi}{\pi - 2t}}$

$$\lim_{t \to 0} \frac{\cos\left(\frac{\pi}{2} - t\right)}{\frac{2\pi}{\pi - 2t} - 2}$$

$$= \lim_{t \to 0} \frac{(\pi - 2t)\cos\left(\frac{\pi}{2} - t\right)}{2\pi - 2(\pi - 2t)}$$ $\sin t$

$$= \lim_{t \to 0} \frac{(\pi - 2t)\sin t}{4t}$$

$$= \lim_{t \to 0} \frac{\sin t}{t} \cdot \lim_{t \to 0} \left(\frac{\pi - 2t}{4}\right)$$ $\frac{\pi}{4}$

$$= \frac{\pi}{4}$$

13. $\lim_{n\to\infty} (n!)^{\frac{1}{n^2}}$

$= \lim_{n\to\infty} e^{\ln (n!)^{\frac{1}{n^2}}}$

$= \lim_{n\to\infty} e^{\frac{\ln(n!)}{n^2}}$

$n(n-1)(n-2)\cdots(1)$

$\ln(n!) = \ln(n \cdot n-1 \cdot n-2 \cdot \cdots 1)$

$= \ln n + \ln n-1 + \cdots + \ln(1)$

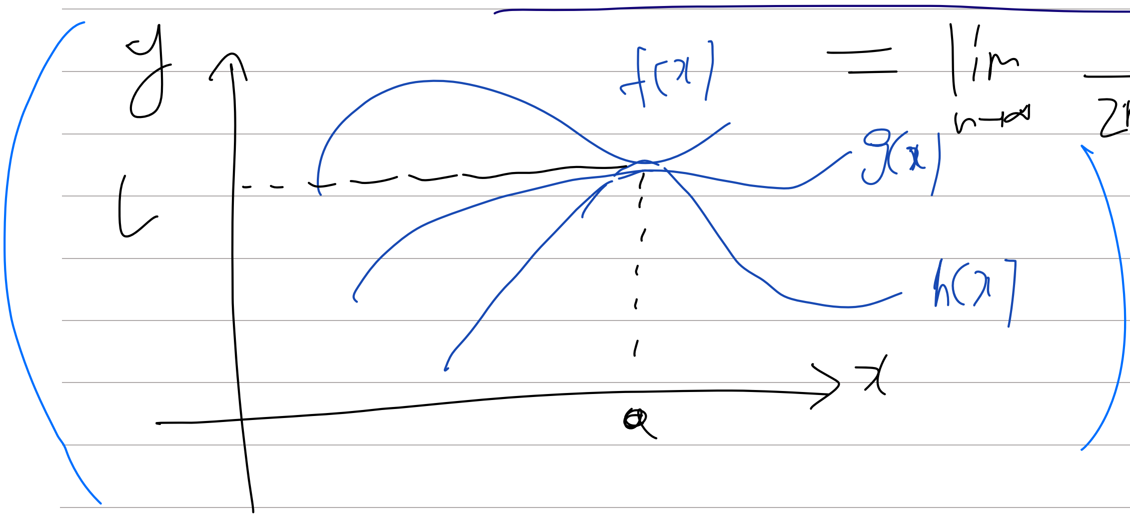
M1 $\lim_{n\to\infty} e^{\frac{\ln n + \ln(n-1) + \cdots + \ln(1)}{n^2}}$

$0 \qquad\qquad 0 \qquad\qquad 0$

$\frac{\ln n}{n^2} > \frac{\ln(n-1)}{n^2} > \frac{\ln(n-2)}{n^2} \cdots > \frac{\ln(1)}{n^2}$

$\lim_{n\to\infty} \frac{\ln(n)}{n^2} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{2n}$

$= \lim_{n\to\infty} \frac{1}{2n^2} = 0$



$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

$$\Rightarrow \underline{\lim_{x\to a} g(x) = L} \; //$$

$$\lim_{n\to\infty} e^{\cancel{\frac{\ln n}{n^2}} \to 0 + \cancel{\frac{\ln(n-1)}{n^2}} \to 0 + \cdots + \cancel{\frac{\ln(1)}{n^2}} \to 0}$$

$$= e^{0} = \boxed{1} \; //$$

M2

$$\frac{\ln(n!)}{n^2} = \frac{\ln(n) + \ln(n-1) + \cdots + \ln(1)}{n^2}$$

squeeze

$$\boxed{\frac{\ln(1) + \ln(1) + \cdots + \ln(1)}{n^2}} \le \boxed{\frac{\ln(n) + \ln(n-1) + \cdots + \ln(1)}{n^2}} \to 0$$

$$\parallel$$

$$\frac{0}{n^2} = 0 \qquad \le \boxed{\frac{\ln(n) + \ln(n) + \cdots + \ln(n)}{n^2}} \; /\!/ \to 0$$

$$\lim_{n\to\infty} \frac{n\ln n}{n^2} = \lim_{n\to\infty} \frac{\ln n \to \infty}{n \to \infty}$$

$$\overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$\rightsquigarrow \lim_{n\to\infty} e^{\boxed{\frac{\ln(n!)}{n^2}} \to 0} = e^0 = 1$$

$$\frac{\ln(1)+\cdots\ln(1)}{n^2} \le \frac{\ln(n)+\cdots+\ln(1)}{n^2} \le \frac{\ln(n)+\cdots+\ln(n)}{n^2}$$

$$\lim_{n\to\infty} \frac{0}{n^2} = 0 \quad \& \quad \lim_{n\to\infty} \frac{n\ln n}{n^2} = 0$$

squeeze

$$\lim_{n\to\infty} \frac{\ln(n!)}{n^2} = 0$$

//

$$\lim_{n \to 0} (1+n)^{\frac{1}{n}} = e$$

$$\lim_{n \to 0} \boxed{e^{\ln(1+n)^{\frac{1}{n}}}} = \lim_{n \to 0} e^{\frac{\ln(1+n)}{n}}$$

$$= \lim_{n \to 0} e^{\frac{1}{1(n+1)}} = e^{1} = e$$