1. $\lim_{x\to a}\frac{x^3-a^3}{x^2-a^2}=3$ 이라고 할 때, $\lim_{x\to a}\frac{x^3-ax^2+a^2x-a^3}{x-a}$ 의 값을 구하시오. **[10 pts]**

$$\lim_{x\to a}\frac{x^3-a^3}{x^2-a^2} = \lim_{x\to a}\frac{\cancel{(x-a)}(x^2+ax+a^2)}{\cancel{(x-a)}(x+a)}$$

$$= \lim_{x\to a}\frac{(x^2+ax+a^2)}{(x+a)} = 3$$

$$\rightsquigarrow \frac{a^2+a^2+a^2}{2a} = \frac{3}{2}a = 3, \quad \underline{a=2}$$

$$\lim_{x\to a}\frac{x^3-ax^2+a^2x-a^3}{x-a}$$

$$= \lim_{x\to a}\frac{\cancel{(x-a)}(x^2+a^2)}{\cancel{x-a}}$$

$$= \lim_{x\to a}(x^2+a^2) = \lim_{x\to 2}(x^2+4) = \underline{8}$$

2. $f(x)=x^2-3x$ 일 때, $\lim_{x\to-\infty}\{\sqrt{f(x)}-\sqrt{f(-x)}\}$ 의 값을 구하시오. **[10 pts]**

$$\lim_{x\to-\infty}\left\{\sqrt{f(x)}-\sqrt{f(-x)}\right\}$$

$x=-t,$

$$\lim_{t\to\infty}\left\{\sqrt{f(-t)}-\sqrt{f(t)}\right\}$$

$$=\lim_{t\to\infty}\left\{\sqrt{t^2+3t}-\sqrt{t^2-3t}\right\}$$

$$=\lim_{t\to\infty}\frac{\left(\sqrt{t^2+3t}-\sqrt{t^2-3t}\right)\left(\sqrt{t^2+3t}+\sqrt{t^2-3t}\right)}{\sqrt{t^2+3t}+\sqrt{t^2-3t}}$$

$$=\lim_{t\to\infty}\frac{(t^2+3t)-(t^2-3t)}{\sqrt{t^2+3t}+\sqrt{t^2-3t}}$$

$$=\lim_{t\to\infty}\frac{6t}{\sqrt{t^2+3t}+\sqrt{t^2-3t}}$$

$$=3$$

3. 두 함수 $f(x), g(x)$가 $\lim_{x\to1} f(x)=\infty$, $\lim_{x\to1}\{2f(x)-g(x)\}=2$ 을 만족할 때,

$\lim_{x\to1}\dfrac{f(x)-2g(x)}{4f(x)-g(x)}$ 의 값을 구하시오. **[10 pts]**

$\lim_{x\to1} f(x)=\infty$, $\lim_{x\to1}\{2f(x)-g(x)\}=2$

$2f(x)-g(x) \triangleq h(x)$ 즉, $\lim_{x\to1} h(x)=2$

$\Leftrightarrow g(x)=2f(x)-h(x)$

$$\lim_{x\to1}\frac{f(x)-2g(x)}{4f(x)-g(x)}$$

$$=\lim_{x\to1}\frac{f(x)-2\{2f(x)-h(x)\}}{4f(x)-2f(x)+h(x)}$$

$$=\lim_{x\to1}\frac{-3f(x)+2h(x)}{2f(x)+h(x)}$$

$$=\lim_{x\to1}\frac{-3+2\cdot\frac{h(x)}{f(x)}}{2+\frac{h(x)}{f(x)}}$$

$\therefore \lim_{x\to1}\dfrac{h(x)}{f(x)}=0$

이므로,

$$\lim_{x\to1}\frac{-3+2\cdot\frac{h(x)}{f(x)}}{2+\frac{h(x)}{f(x)}}$$

$$=-\frac{3}{2}$$

4. 극한 $\lim_{x \to 0} \frac{\tan(\sin \pi x)}{x}$ 의 값을 계산하면? [10 pts]

$$\lim_{x \to 0} \frac{\tan(\sin \pi x)}{x}$$

$$= \lim_{x \to 0} \frac{\tan(\sin \pi x)}{\sin \pi x} \times \frac{\sin \pi x}{\pi x} \times \pi = \pi$$

1 1

5. 극한 $\lim_{x\to 0} \frac{1}{(1-\sin 2x)^{\cot x}}$ 의 극한값을 구하면? [10 pts]

$$\lim_{x\to 0} (1-\sin 2x)^{\cot x}$$

$$= \lim_{x\to 0} e^{\ln (1-\sin 2x)^{\cot x}}$$

$$= \lim_{x\to 0} e^{\frac{\cancel{\cos x}^{1} \ln (1-\sin 2x)}{\sin x}}$$

$$= \lim_{x\to 0} e^{\frac{\ln(1-\sin 2x)}{\sin x}}$$

$\frac{0}{0}$

$$= \lim_{x\to 0} e^{\frac{-2\cancel{\cos 2x}^{1}}{\cancel{\cos x}^{1}(1-\sin 2x)}}$$

$$= \lim_{x\to 0} e^{-2} = \frac{1}{e^2} \neq 0$$

즉 $\lim_{x\to 0} \frac{1}{(1-\sin 2x)^{\cot x}}$

$$= \frac{\lim_{x\to 0} 1}{\lim_{x\to 0} (1-\sin 2x)^{\cot x}} = \frac{1}{1/e^2} = \underline{e^2}$$

6. 극한 $\lim_{x \to \infty} \left(\frac{x+\sqrt{5}}{x-\sqrt{5}} \right)^{\sqrt{5}x}$ 의 값을 구하면? [10 pts]

$$\lim_{x \to \infty} \left(\frac{x+\sqrt{5}}{x-\sqrt{5}} \right)^{\sqrt{5}x}$$

$$= \lim_{x \to \infty} e^{\ln\left(\frac{x+\sqrt{5}}{x-\sqrt{5}} \right)^{\sqrt{5}x}}$$

$$= \lim_{x \to \infty} e^{\sqrt{5}x \{\ln(x+\sqrt{5}) - \ln(x-\sqrt{5})\}}$$

$$= \lim_{x \to \infty} e^{\sqrt{5} \cdot \frac{\{\ln(x+\sqrt{5}) - \ln(x-\sqrt{5})\}}{\frac{1}{x}}}$$

$$\overset{\text{로피}}{=} \lim_{x \to \infty} e^{\sqrt{5} \cdot \frac{\frac{1}{x+\sqrt{5}} - \frac{1}{x-\sqrt{5}}}{-\frac{1}{x^2}}}$$

$$= \lim_{x \to \infty} e^{\sqrt{5} \frac{\frac{-2\sqrt{5}}{x^2-5}}{-\frac{1}{x^2}}}$$

$$= \lim_{x \to \infty} e^{\frac{2\sqrt{5}x^2}{x^2-5} \cdot \sqrt{5}}$$

$$= \lim_{x \to \infty} e^{\frac{10x^2}{x^2-5}} = e^{10}$$

7. a가 양수일 때, $\lim_{x \to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$ 의 값을 구하면? **[10 pts]**

$$\lim_{x \to a} \frac{\sqrt{2a^3x - x^4} - a \cdot \sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

$\frac{0}{0}$

$$= \lim_{x \to a} \frac{\frac{2a^3 - 4x^3}{2\sqrt{2a^3x - x^4}} - a\left(\frac{1}{3}\right)(a^2x)^{-\frac{2}{3}}(a^2)}{-\frac{1}{4}(ax^3)^{-\frac{3}{4}}(3ax^2)}$$

$$= \lim_{x \to a} \frac{\frac{-2a^3}{2\sqrt{a^4}} - \frac{1}{3}a^3(a^3)^{-\frac{2}{3}}}{-\frac{3}{4}(a^4)^{-\frac{3}{4}}(a^3)}$$

$$= \lim_{x \to a} \frac{-a - \frac{1}{3} \cdot a}{-\frac{3}{4}}$$

$$= -\frac{4}{3}a \times \left(-\frac{4}{3}\right) = \frac{16}{9}a$$

8. 로피탈 정리를 사용하지 않고 $\lim\limits_{x\to\sqrt{2}}\dfrac{(x^2+x)\sin(x-\sqrt{2})}{x^2-2}$ 를 계산하시오. **[10 pts]**

$$\lim_{x\to\sqrt{2}}\frac{(x^2+x)\sin(x-\sqrt{2})}{x^2-2}$$

$$=\lim_{x\to\sqrt{2}}\frac{(x^2+x)\sin(x-\sqrt{2})}{(x+\sqrt{2})(x-\sqrt{2})}$$

$$=\lim_{x\to\sqrt{2}}\frac{x^2+x}{(x+\sqrt{2})}\cdot\cancelto{1}{\lim_{x\to\sqrt{2}}\frac{\sin(x-\sqrt{2})}{(x-\sqrt{2})}}$$

$$=\lim_{x\to\sqrt{2}}\frac{x^2+x}{x+\sqrt{2}}=\frac{2+\sqrt{2}}{2\sqrt{2}}=\frac{2+2\sqrt{2}}{4}$$

$$=\underline{\frac{1}{2}(1+\sqrt{2})}$$

9. 극한 $\lim_{x\to0}\frac{\sqrt{2+\tan x}-\sqrt{2+\sin x}}{x^3}$ 의 값을 계산하시오. **[10 pts]**

(로피탈 X)

$$\lim_{x\to0}\frac{\sqrt{2+\tan x}-\sqrt{2+\sin x}}{x^3}$$

$$=\lim_{x\to0}\frac{\left(\sqrt{2+\tan x}-\sqrt{2+\sin x}\right)\left(\sqrt{2+\tan x}+\sqrt{2+\sin x}\right)}{x^3\left(\sqrt{2+\tan x}+\sqrt{2+\sin x}\right)}$$

$$=\lim_{x\to0}\frac{(2+\tan x)-(2+\sin x)}{x^3\left(\sqrt{2+\tan x}+\sqrt{2+\sin x}\right)}$$

$$=\lim_{x\to0}\frac{\tan x-\sin x}{x^3\left(\sqrt{2+\tan x}+\sqrt{2+\sin x}\right)}$$

$$\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$$

$$=\lim_{x\to0}\frac{\frac{1}{\cos x}(\sin x-\sin x\cos x)}{x^3}$$

$$=\lim_{x\to0}\frac{\sin x(1-\cos x)}{\cancel{\cos x}\ ?!\ x^3}$$

$$=\lim_{x\to0}\frac{\sin x(1-\cos x)(1+\cos x)}{x^3(1+\cos x)}$$

$$=\frac{1}{2}\lim_{x\to0}\frac{\sin x\cdot\sin^2 x}{x^3}=\frac{1}{2}\lim_{x\to0}\left(\frac{\sin x}{x}\right)^3$$

$$=\frac{1}{2}.$$

$$\lim_{x\to0} \frac{1}{\sqrt{2+\tan x}+\sqrt{2+\sin x}} = \frac{1}{2\sqrt{2}}$$

$\because$ $$\lim_{x\to0} \frac{\tan x-\sin x}{x^3\left(\sqrt{2+\tan x}+\sqrt{2+\sin x}\right)}$$

$$= \lim_{x\to0} \frac{\tan x-\sin x}{x^3} \cdot \lim_{x\to0} \frac{1}{\sqrt{2+\tan x}+\sqrt{2+\sin x}}$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$$

10. 극한 $\lim_{x\to\infty}\left\{\frac{1}{e}\left(1+\frac{1}{x}\right)^{x}\right\}^{x}$ 의 값을 계산하시오. [10 pts]

$$\lim_{x\to\infty}\left\{\frac{1}{e}\left(1+\frac{1}{x}\right)^{x}\right\}^{x}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x^2}\cdot e^{-x}$$

$$=\lim_{x\to\infty} e^{\ln\left\{\left(1+\frac{1}{x}\right)^{x^2}\cdot e^{-x}\right\}}$$

$$=\lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^{x^2}+\ln e^{-x}}$$

$$=\lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^{x^2}-x}$$

$$=\lim_{x\to\infty} e^{\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x^2}}-x}$$

$$=\lim_{x\to\infty} e^{\frac{\ln\left(1+\frac{1}{x}\right)-\frac{1}{x}}{\frac{1}{x^2}}}$$

$\frac{1}{x}=t$

$$\lim_{t\to0} e^{\frac{\ln(1+t)-t}{t^2}}$$

$$\overset{\frac{0}{0}}{=}\lim_{t\to0} e^{\frac{\frac{1}{1+t}-1}{2t}}$$

$$=\lim_{t\to0} e^{\frac{-\cancel{t}}{2\cancel{t}(1+t)}}$$

$$=\lim_{t\to0} e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$$