Review

집합, 명제

$\{(x,y) \mid x^2+y^2=1\}$. $\ni (1,0), (0,1), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), \dots$

$\underbrace{(x,y)}_{\text{대상}}$ $\underbrace{x^2+y^2=1}_{\text{조건}}$

"3은 1보다 작다"

"$x=1$" (x의 값에 따라 참/거짓 바뀜 => 조건).

"p이면 q이다" (p, q: 조건).

ex) 소수이면 홀수이다 (소수 → 홀수).

↳ 2 (소수이지만 홀수 X, 반례).

정량자. ex)

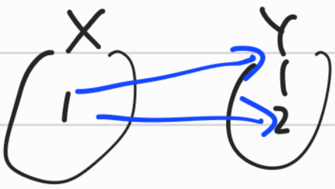
$\forall$ (for all) $\forall x \in \mathbb{R},\ x^2 \geq 0$.

For all x in $\mathbb{R}$, $x^2 \geq 0$.

$\exists$ (There exists) $f: X \to Y$. $\forall x \in X, \exists y \in Y$ s.t. $f(x)=y$.

For all x in X, there exists y in Y such that $f(x)=y$.

$\exists!$ (There exists unique) $f: X \to Y$. $\forall x \in X, \exists! y \in Y$ s.t. $f(x)=y$.



[Input] → [Output].

* 증명의 방법. $p \to q$.

1. Direct Proof

WTS: $p \to q$ $\quad p \to r \to s \to t \to q$. $\therefore p \to q$.

want To Show

ex) 짝수의 제곱은 짝수다.

$n = 2k \quad (k \in \mathbb{Z})$.

$\to n^2 = (2k)^2 = 4k^2 = 2(\underbrace{2k^2}_{\in \mathbb{Z}}) = 2k' \quad (k' = 2k^2)$.

$\therefore$ 짝수^2 = 짝수.

2. 귀류법.

WTS: $p \to q$.

가정: $p \to \sim q$. (not q). $\text{notnot } q = q$.

ex) $\sqrt{2}$: 무리수.

↳ 실수 중에서, 유리수가 아닌것.

↳ $\frac{q}{p}$ ($p \neq 0$, p&q=서로소).

"귀류법" $\sqrt{2}$ 가 유리수라고 가정하자.

$\exists\, p, q \in \mathbb{Z}$, $p \neq 0$, p와 q는 서로소 s.t.

$\sqrt{2} = \frac{q}{p}$. $\Rightarrow$ $2 = \frac{q^2}{p^2}$ $\Rightarrow$ $2p^2 = q^2$.

$\in \mathbb{Z}$

$\Rightarrow \exists\, \ell \in \mathbb{Z}, \ell \neq 0$ s.t. $q = 2\ell$.

$\Rightarrow 2p^2 = (2\ell)^2 = 4\ell^2$

$\Rightarrow p^2 = 2\ell^2$.

$\in \mathbb{Z}$

$\Rightarrow \exists\, m \in \mathbb{Z}, m \neq 0$. s.t. $p = 2m$.

$\therefore$ $p = 2m$, $q = 2\ell$. p와 q가 2라는 공약수 가짐.

$\rightarrow\leftarrow$ p와 q가 서로소.

모순.

$\therefore$ $\sqrt{2}$는 무리수!

3. 대우를 이용한 증명.

$p \to q$ 대우? $\sim q \to \sim p$.

T   T

F   F.

ex) 제곱이 홀수인 수는 홀수이다.

pf) 대우: 짝수이면 제곱이 짝수이다.

$n = 2k$. $\Rightarrow$ $n^2 = (2k)^2 = 4k^2$.

4. 수학적 귀납법

$P(n)$. WTS: $\forall n \in \mathbb{N}$, $P(n)$이 참.

pf) ① $n=1$일 때 참.

② $n=k$일 때 참 $\Rightarrow$ $n=k+1$일때 참.

* 선형대수학.

1~9강

선형대수학 기초

공학수학.

비수학과-선대

계산, 응용

10강 ~

선형대수학 심화.

Serge Lang. → 개념

수학과 선대.

증명, 이해.

① 시간없다X.

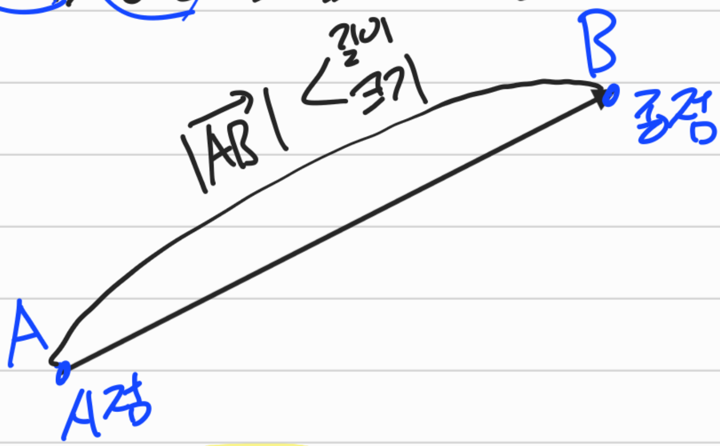
② 어떻게 공부X.

③ 선대 진입장벽.

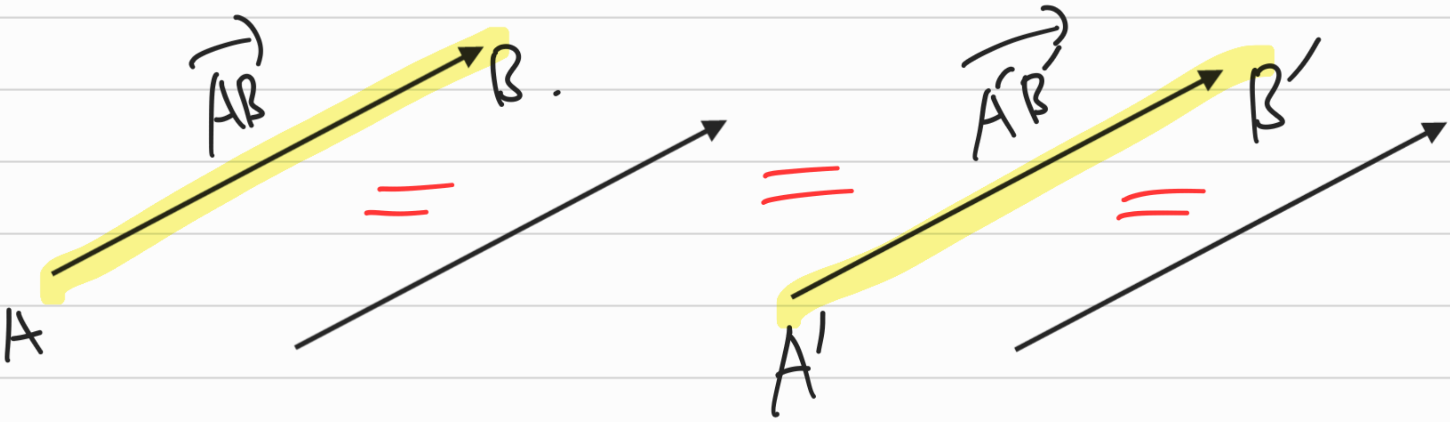
벡터

1. 벡터의 정의.

벡터 : 크기, 방향 갖고있는 물리량.

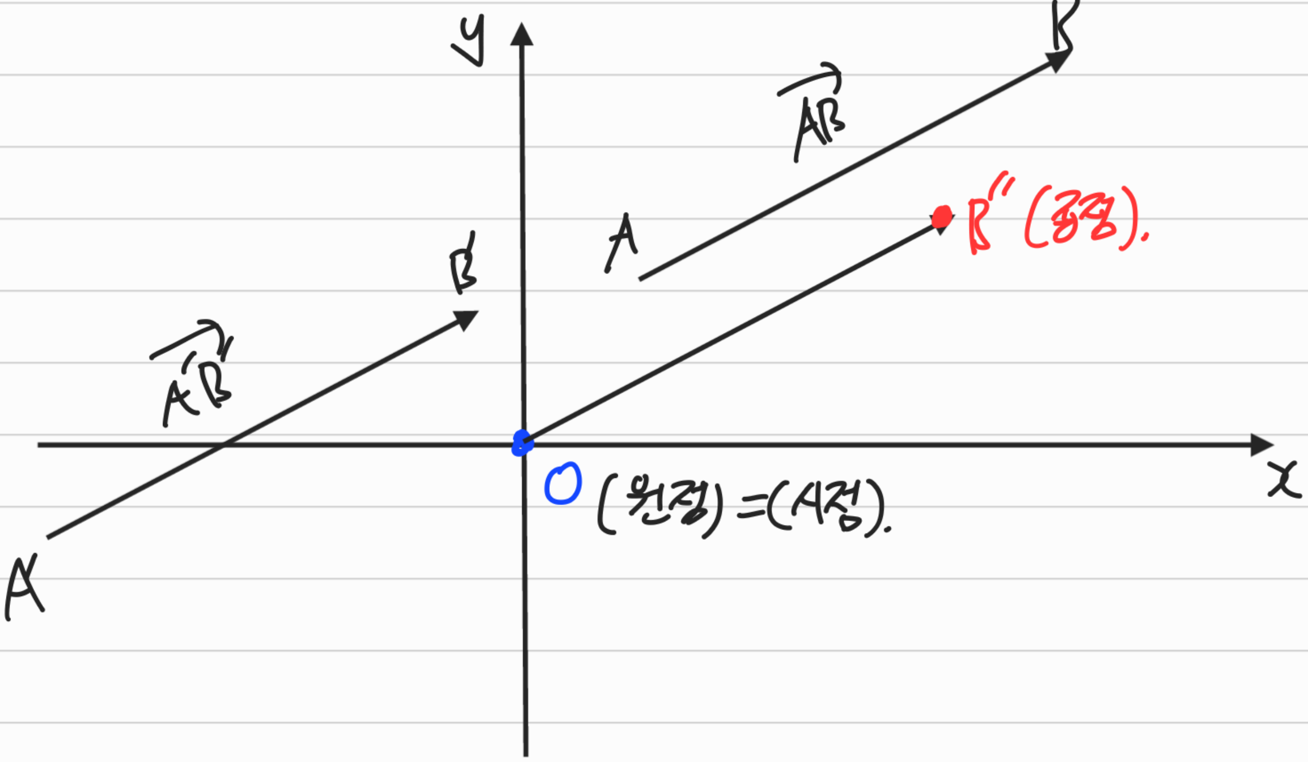


2. 서로 같은 벡터. [크기 방향.



3. Component form.

이차원 평면 (좌표평면).



Before Component Form : 시점, 종점 → 벡터.

After Component Form : 종점 → 벡터. (시점이 항상 원점)

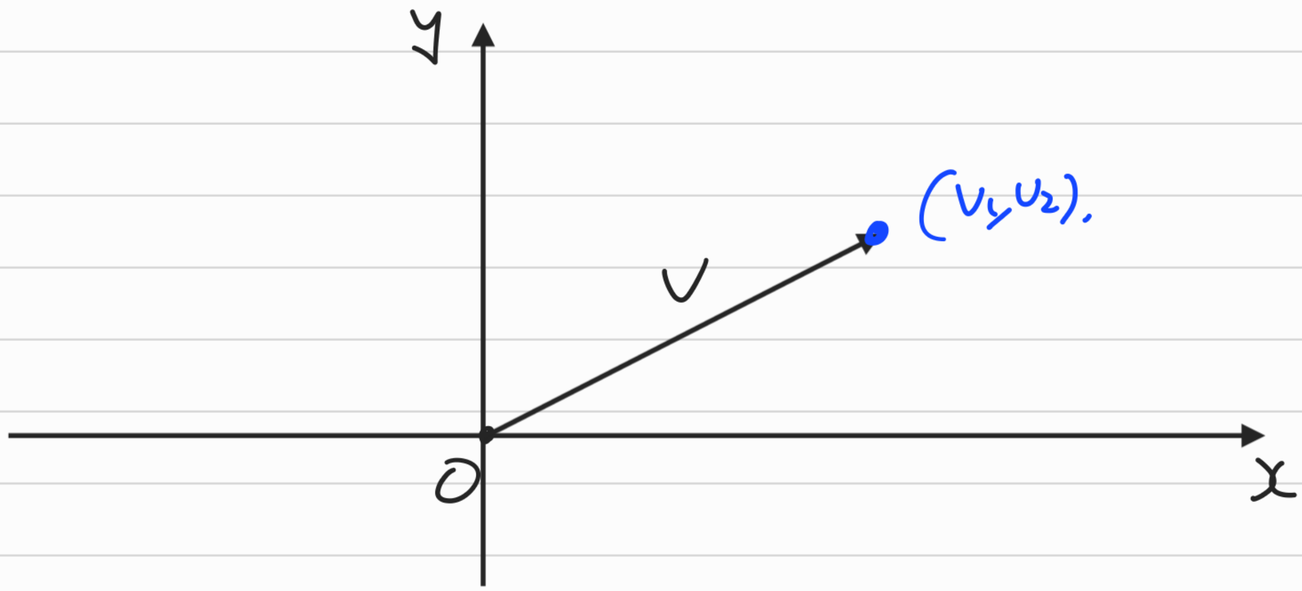
ex) ① 벡터1의 종점 ≠ 벡터2의 종점

⇒ 벡터1 ≠ 벡터2.

② 벡터1의 종점 = 벡터2의 종점

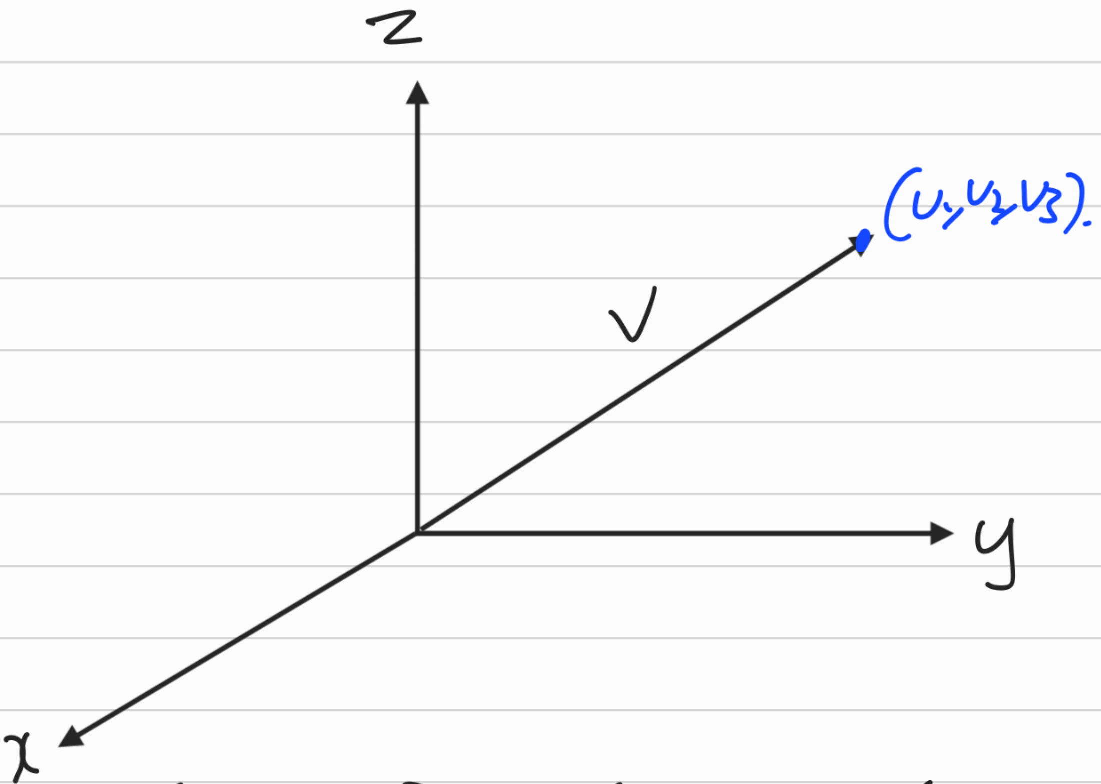
⇒ 벡터1 = 벡터2.

*이차원 평면.



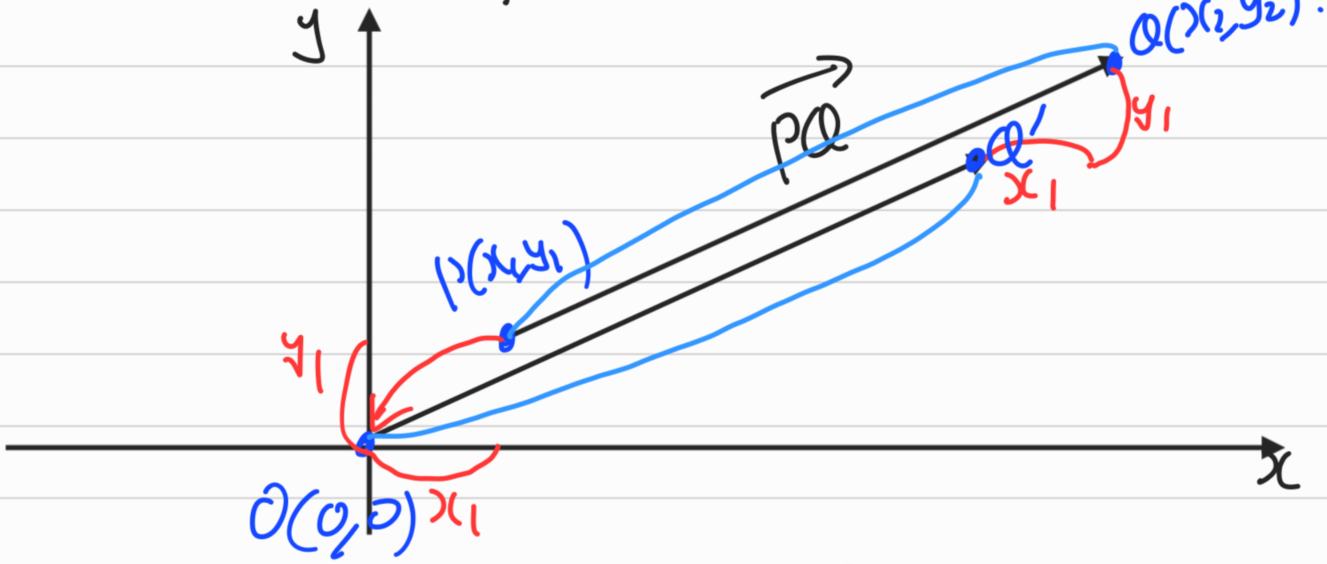
벡터 v의 Component form $= \langle v_1, v_2 \rangle = (v_1, v_2)$.

*삼차원 공간.



벡터 v의 Component form $= \langle v_1, v_2, v_3 \rangle = (v_1, v_2, v_3)$.

4. 두 점을 잇는 벡터의 Component Form.



$P(x_1, y_1) \rightarrow O(0,0)$ $(-x_1, -y_1)$만큼 평행이동

$Q(x_2, y_2) \rightarrow Q'(x_2 - x_1, y_2 - y_1)$. $(-x_1, -y_1)$ 만큼 평행이동

$\therefore P(x_1, y_1),\ Q(x_2, y_2)$ $|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \overrightarrow{PQ} = (x_2 - x_1, y_2 - y_1)$.   크기

ex) v: $(2,1) \rightarrow (0,0)$.

v의 Component form $v = \langle 0-2, 0-1 \rangle$

$= \langle -2, -1 \rangle$.

5. 영벡터 ($\mathbb{R}^3$)

$0 = \langle 0, 0, 0 \rangle$.

① 길이가 0인 유일한 벡터.

② 방향이 없는 유일한 벡터.

6. 스칼라.

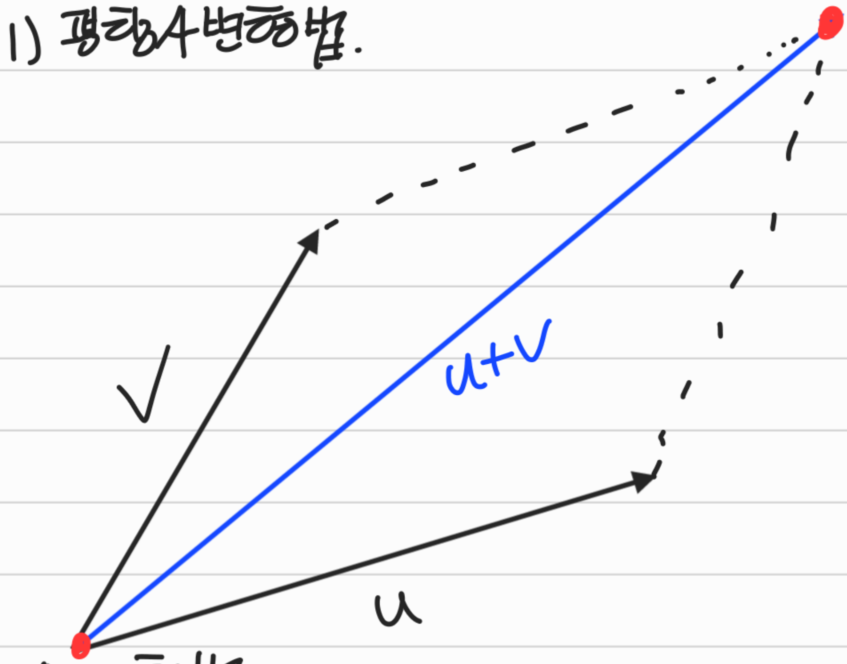
하나의 수치만으로 완전히 표현되는 양.

ex) $\mathbb{R}$, $\mathbb{C}$.

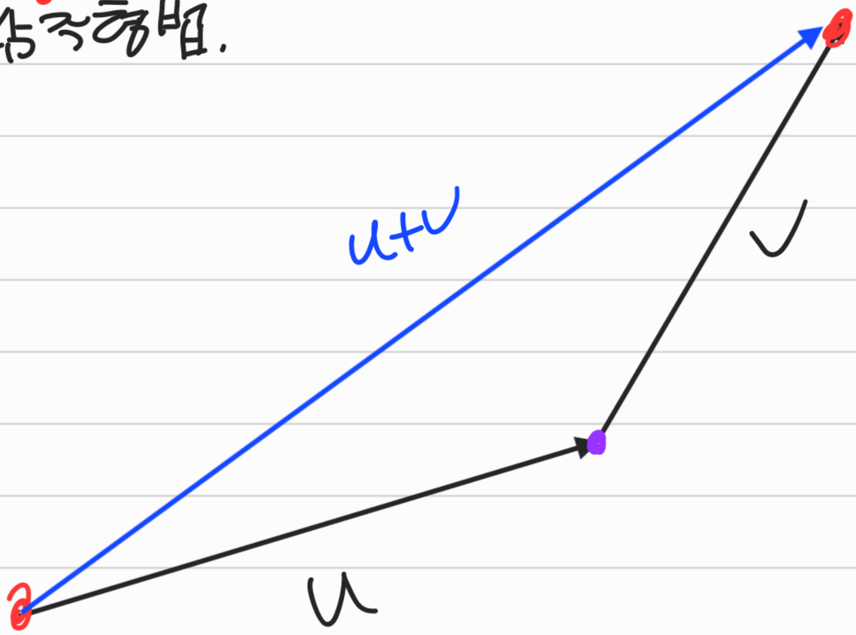
벡터의 연산

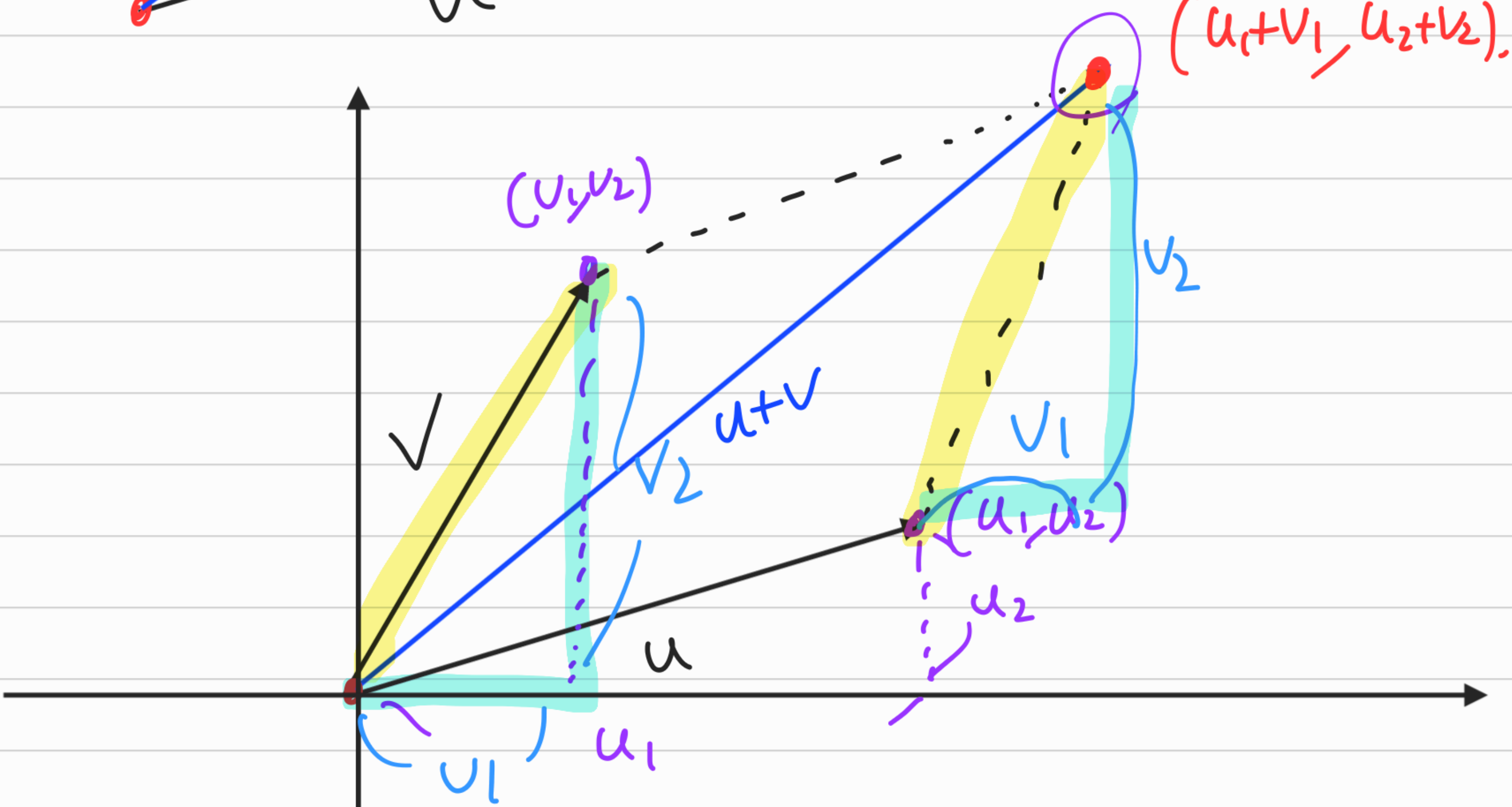
* 벡터 + 벡터.

1) 평행사변형법.



2) 삼각형법.



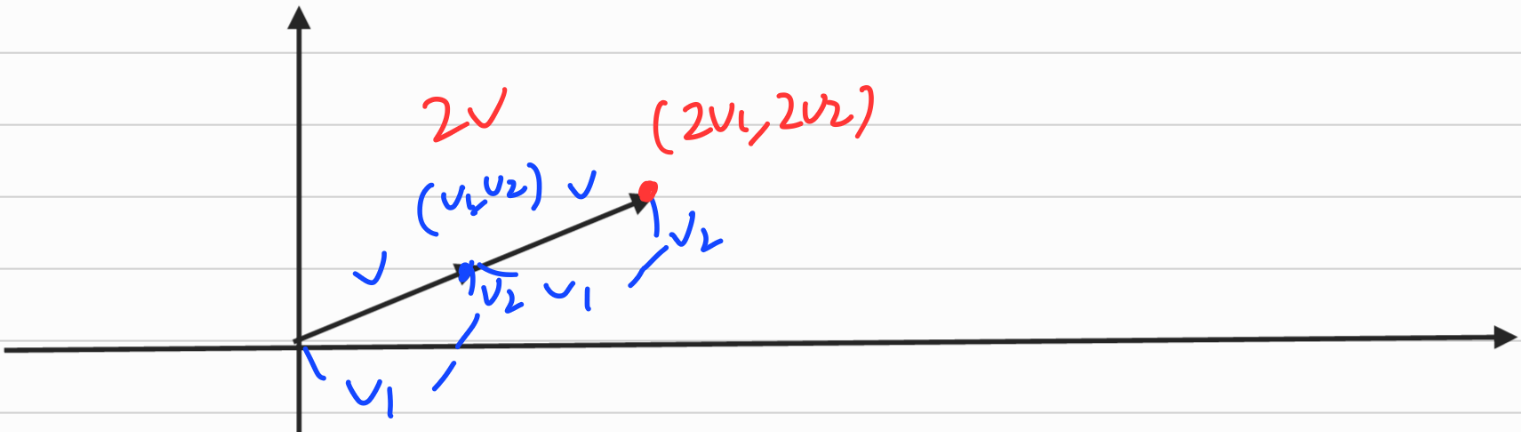


$\therefore (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$.

ex) $(3,2) + (1,4) = (4,6)$.

2) 벡터의 스칼라 곱. $\quad f(x) = x^2$.

벡터 × 스칼라 → 벡터. $\quad 2f(x) = 2x^2$.



$v = (v_1, v_2)$. $\quad kv = k(v_1, v_2) = (kv_1, kv_2)$.

$v = (v_1, v_2)$. $\quad -v = (-1)v = (-1)(v_1, v_2) = (-v_1, -v_2)$.

$v + (-v) = (v_1, v_2) + (-v_1, -v_2) = (v_1 - v_1, v_2 - v_2)$

$= (0, 0) = 0$.

$(-1)v$.

* 0: 스칼라 $\quad$ u=벡터. $\quad u = \langle u_1, u_2 \rangle$

$0u = 0\langle u_1, u_2 \rangle = \langle 0u_1, 0u_2 \rangle = \langle 0, 0 \rangle = 0$.

⇒ 어떤 벡터의 0배는 영벡터.

ex) u=벡터 $\quad -2u$? $\quad$ [① 크기=u의 2배 ② 방향=u와 정반대.

* $-2u = (-2)u = (2)(-1)u = 2(-u)$.

* ku의 크기.

$u = \langle u_1, u_2 \rangle \Rightarrow ku = \langle ku_1, ku_2 \rangle.$

$$|u| = \sqrt{u_1^2 + u_2^2} \qquad |ku| = \sqrt{(ku_1)^2 + (ku_2)^2}$$
$$= \sqrt{k^2(u_1^2 + u_2^2)}$$
$$= |k|\sqrt{u_1^2 + u_2^2}$$
$$= |k|\,|u|.$$

∴ u의 k배의 크기는 u의 크기의 $|k|$배다.

* 벡터의 차.

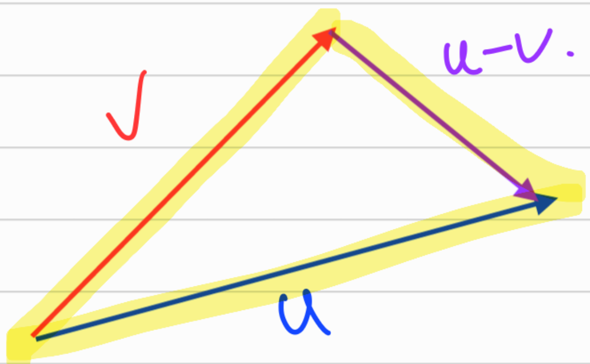
$u - v = u + (-v).$

$u = \langle u_1, u_2 \rangle, \quad v = \langle v_1, v_2 \rangle.$

$$\Rightarrow u - v = u + (-v) = \langle u_1, u_2 \rangle + (-1)v$$
$$= \langle u_1, u_2 \rangle + (-1)\langle v_1, v_2 \rangle$$
$$= \langle u_1, u_2 \rangle + \langle -v_1, -v_2 \rangle.$$
$$= \langle u_1 - v_1, u_2 - v_2 \rangle.$$

$u - v$: v와 더했을때 u가 되는 벡터.

$(u-v) + v = u \underbrace{- v + v} = u + 0 = u.$



2. 벡터의 연산의 성질.

u, v, w = 벡터, a & b = 스칼라.

1) $u+v = v+u$. (교환법칙).

pf) $u = \langle u_1, u_2 \rangle$, $v = \langle v_1, v_2 \rangle$.

$$\begin{aligned} u+v &= \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle \\ &= \langle u_1+v_1, u_2+v_2 \rangle \\ &= \langle v_1+u_1, v_2+u_2 \rangle \\ &= \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle \\ &= v+u. \end{aligned}$$

2) $(u+v)+w = u+(v+w)$ (결합법칙).

pf) $u = \langle u_1, u_2 \rangle$, $v = \langle v_1, v_2 \rangle$, $w = \langle w_1, w_2 \rangle$. Check!

3) $u + 0 = u$. (영벡터, 덧셈에 대한 항등원)

pf) $u = \langle u_1, u_2 \rangle$, $0 = \langle 0, 0 \rangle$. Check!

4) $u + \boxed{(-u)} = 0$. ($-u$ = u의 덧셈에 대한 역원)

pf) $u = \langle u_1, u_2 \rangle$, $-u = (-1)u = \langle -u_1, -u_2 \rangle$. Check!

5) $0u = 0$.

6) $1u = u$.

7) $a(bu) = (ab)u$.

8) $\boxed{a}(u+v) = au + av$ (분배법칙).

pf) $u = \langle u_1, u_2 \rangle$, $v = \langle v_1, v_2 \rangle$.

$$\begin{aligned} a(u+v) &= a(\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle) \\ &= a(\langle u_1+v_1, u_2+v_2 \rangle) \\ &= \langle a(u_1+v_1), a(u_2+v_2) \rangle \\ &= \langle au_1+av_1, au_2+av_2 \rangle \\ &= \langle au_1, au_2 \rangle + \langle av_1, av_2 \rangle \\ &= a\langle u_1, u_2 \rangle + a\langle v_1, v_2 \rangle. \\ &= au + av. \end{aligned}$$

스칼라.

9) $(a+b)u = au + bu$ (분배법칙)

pf) $u = \langle u_1, u_2 \rangle$. Check!

3. 단위벡터와 표준단위벡터.

단위벡터 (unit vector) : 길이가 1인 벡터.

ex) $u = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$, $v = \langle -1, 0 \rangle$.

표준단위벡터 (standard unit vector) $\mathbb{R}^3$.

$i = \langle 1, 0, 0 \rangle$, $j = \langle 0, 1, 0 \rangle$, $k = \langle 0, 0, 1 \rangle$.

$\forall v \in \mathbb{R}^3$, $v = \langle v_1, v_2, v_3 \rangle$ $(v_i \in \mathbb{R}, i = 1, 2, 3)$.

$$v = \langle v_1, v_2, v_3 \rangle = \langle v_1, 0, 0 \rangle + \langle 0, v_2, 0 \rangle + \langle 0, 0, v_3 \rangle$$
$$= v_1 \langle 1, 0, 0 \rangle + v_2 \langle 0, 1, 0 \rangle + v_3 \langle 0, 0, 1 \rangle$$
$$= \underbrace{v_1}_{v\text{의 } i\text{-component}} i + \underbrace{v_2}_{v\text{의 } j\text{-comp}} j + \underbrace{v_3}_{v\text{의 } k\text{-comp}} k.$$

→ 선형결합, 일차결합 (linear combination)

4. 선형결합, 일차결합.

$v_1, v_2, \ldots, v_n$: 벡터.

$a_1, a_2, \ldots, a_n$: 실수.

$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n$: $v_1, \ldots, v_n$의 선형결합, 일차결합.

ex) v, w $\quad 2v + 3w$: v, w의 선형결합.

* 임의의 벡터 v → 단위벡터 (v와 방향은 같게).

kv [v와 방향은 같음 / 크기 $|k| \|v\|$]

$|kv| = |k||v| = 1$ where $|k| = \frac{1}{|v|}$ (양수). $\therefore k = \frac{1}{|v|}$.

$$v \longrightarrow \frac{v}{|v|}$$

v와 방향은 같으면서 크기가 1인 단위벡터.

* 벡터 × 벡터 → 벡터 [덧셈 / 스칼라곱.

* 벡터 × 벡터 → 스칼라? (내적).

벡터의 내적

1. 벡터의 내적

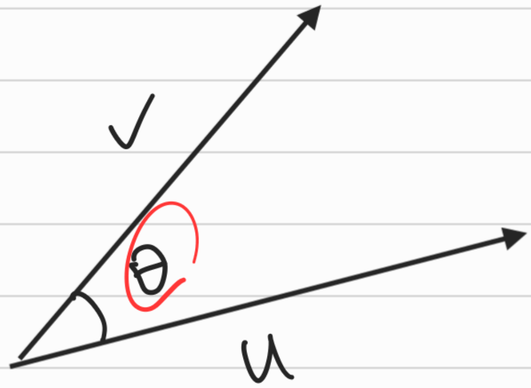
$u = \langle u_1, u_2, u_3 \rangle$, $v = \langle v_1, v_2, v_3 \rangle$.

$u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$.

2. 벡터의 내적과 각도.

u, v : 벡터.

$u \cdot v$: u, v의 내적.



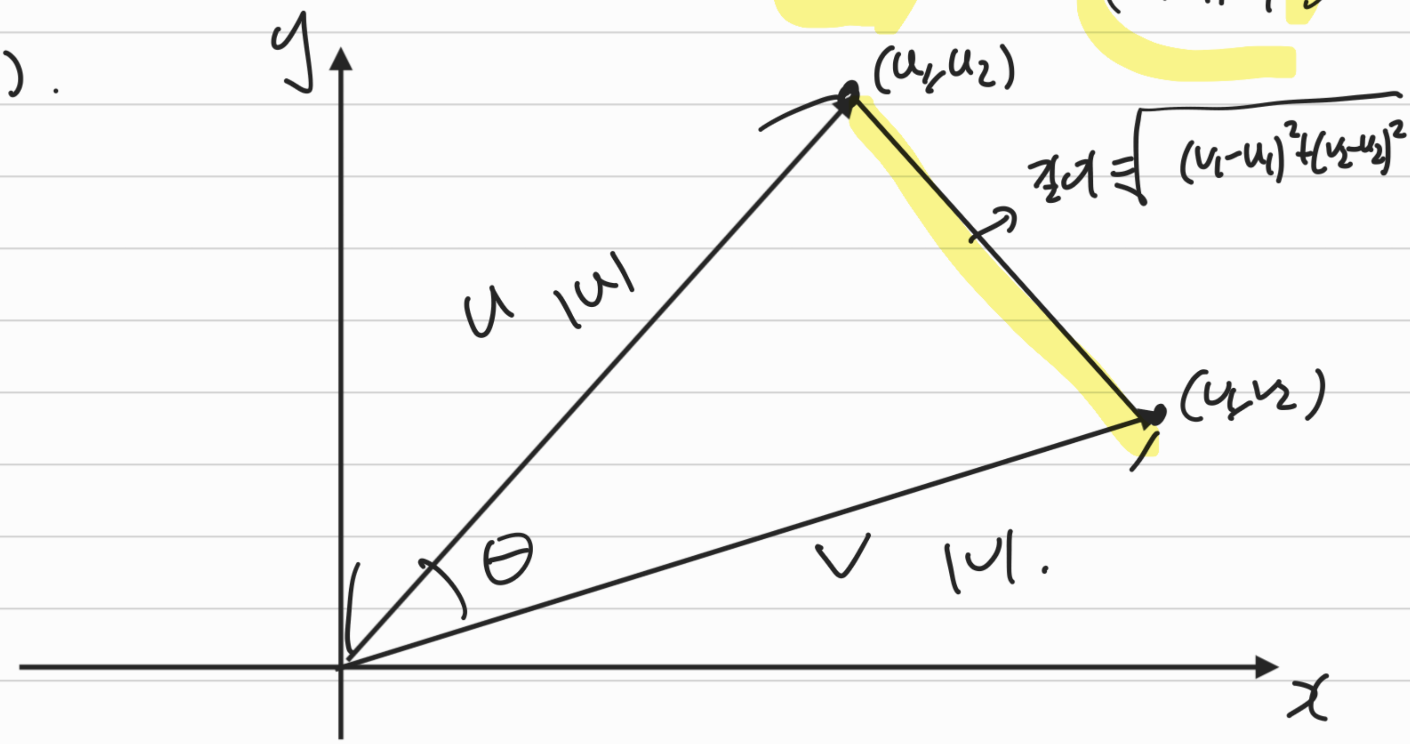
$u \cdot v = |u||v| \cos\theta$.

쉬움 쉬움. 쉬움. 어려움.

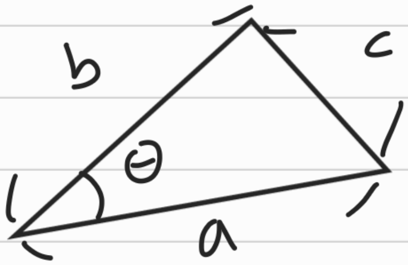
$\cos\theta = \dfrac{u \cdot v}{|u||v|}$.

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{u \cdot v}{|u||v|}\right)$.

pf).



★ 코사인법칙



$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$

$$\Rightarrow (v_1-u_1)^2 + (v_2-u_2)^2.$$

$$= |u|^2 + |v|^2 - 2|u||v|\cos\theta.$$

$$\Rightarrow u_1^2 - 2u_1v_1 + v_1^2 + u_2^2 - 2u_2v_2 + v_2^2$$

$$= |u|^2 + |v|^2 - 2|u||v|\cos\theta.$$

$$\Rightarrow -2u_1v_1 - 2u_2v_2$$

$$= -2|u||v|\cos\theta.$$

$$\Rightarrow u_1v_1 + u_2v_2 = |u||v|\cos\theta.$$

$$\Rightarrow \underset{\|}{u \cdot v} = |u||v|\cos\theta.$$

$|u| = \sqrt{u_1^2 + u_2^2}$

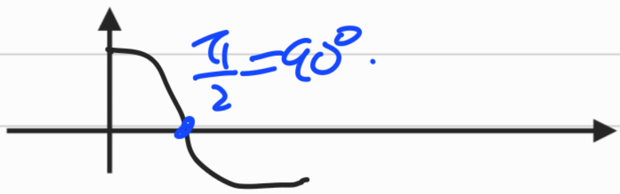
$|v| = \sqrt{v_1^2 + v_2^2}.$

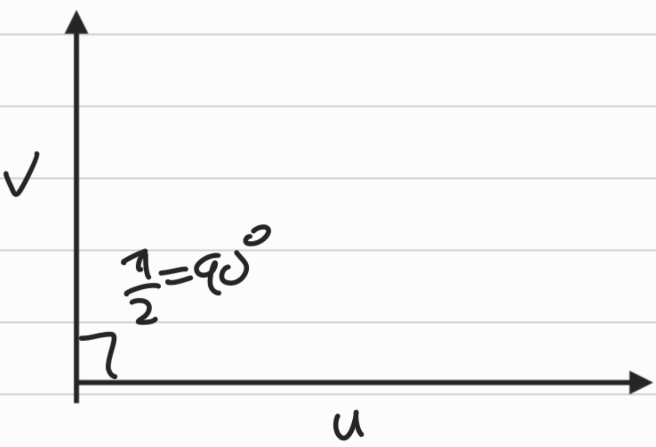
3. 서로 직교하는 벡터.

u, v = 벡터.

$u \cdot v = 0 \Rightarrow u$ & v = 직교 (orthogonal).

↳ $\underbrace{|u|}_{\neq 0}\underbrace{|v|}_{\neq 0}\underbrace{\cos\theta}_{= 0} = 0.$





$u \cdot v = 0$

$\Rightarrow u, v$ 직교!

4. 내적의 성질.

u, v, w = 벡터, c = 스칼라.

1) $u \cdot v = v \cdot u$. (교환법칙)

pf) $u \cdot v = |u||v|\cos\theta$.

$v \cdot u = |v||u|\cos\theta$ //

2) $(cu) \cdot v = u \cdot (cv) = c(u \cdot v)$

스칼라.

pf) $(cu) \cdot v$

$= (c\langle u_1, u_2\rangle) \cdot \langle v_1, v_2\rangle$

$= \langle cu_1, cu_2\rangle \cdot \langle v_1, v_2\rangle$

$= cu_1v_1 + cu_2v_2.$

$= c(u_1v_1 + u_2v_2)$

$= c(u \cdot v)$

3) $u \cdot (v+w) = u \cdot v + u \cdot w$ (분배법칙).

pf) $u = \langle u_1, u_2\rangle$, $v = \langle v_1, v_2\rangle$, $w = \langle w_1, w_2\rangle$.

$u \cdot (v+w) = \langle u_1, u_2\rangle \cdot (\langle v_1, v_2\rangle + \langle w_1, w_2\rangle)$

$= \langle u_1, u_2\rangle \cdot \langle v_1+w_1, v_2+w_2\rangle.$

$= u_1(v_1+w_1) + u_2(v_2+w_2).$

$= u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2.$

$= u_1v_1 + u_2v_2 + u_1w_1 + u_2w_2$

$= \langle u, v\rangle + \langle u, w\rangle.$

4) $u \cdot u = |u|^2.$

pf) $u = \langle u_1, u_2\rangle.$

$\Rightarrow u \cdot v = \langle u_1, u_2\rangle \cdot \langle u_1, u_2\rangle$

$= u_1^2 + u_2^2 = (\sqrt{u_1^2+u_2^2})^2$

$= |u|^2.$

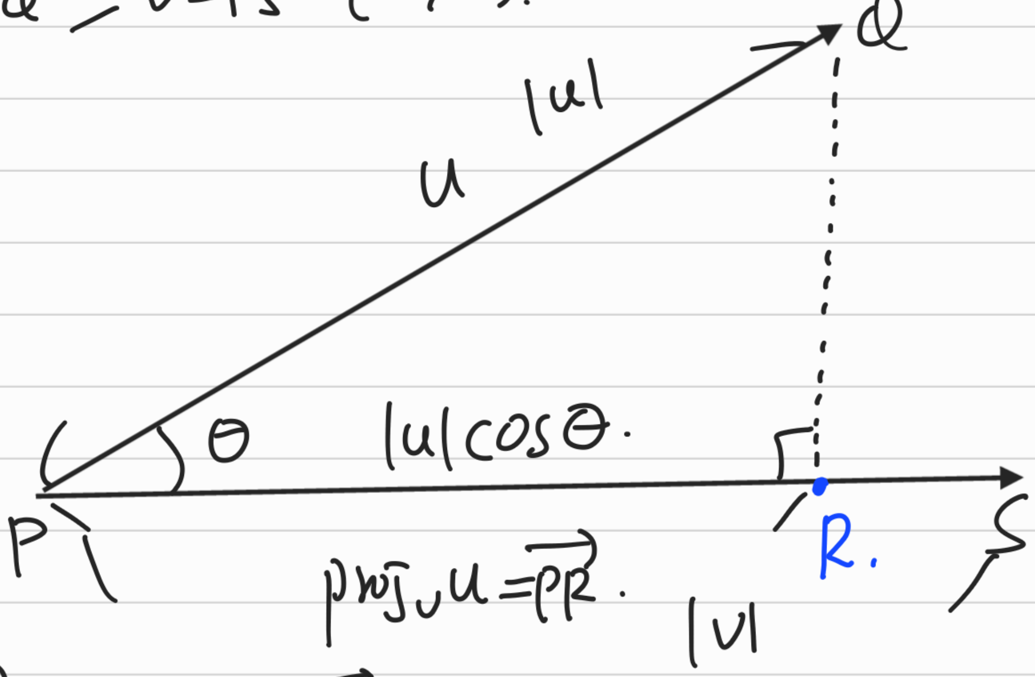
5) $\underset{\text{벡터}}{0} \cdot u = \underset{\text{스칼라}}{0}.$

pf) $0 \cdot u = |0||u|\cos\theta.$

$= 0.$ $(\because |0| = 0).$

5. 벡터의 사영.

$u = \overrightarrow{PQ}$, $v = \overrightarrow{PS}$ $(v \neq 0)$.



$\overrightarrow{PR}$ [방향 : $\overrightarrow{PS}$와 같다. 크기 : $|u|\cos\theta$.

$\dfrac{v}{|v|}$ [방향 = $v = \overrightarrow{PS}$와 같음 크기 = 1

단위.

$$\mathrm{proj}_v u = (|u|\cos\theta)\left(\frac{v}{|v|}\right) = \left(\frac{|u|\cos\theta}{|v|}\right)v.$$

어려움.

Note. $u \cdot v = |u||v|\cos\theta$.

$$\Rightarrow \frac{u \cdot v}{|v|^2} = \frac{|u|\cos\theta}{|v|}$$

$$\therefore \mathrm{proj}_v u = \left(\frac{|u|\cos\theta}{|v|}\right)v = \left(\frac{u \cdot v}{|v|^2}\right)v.$$

쉬움 쉬움.

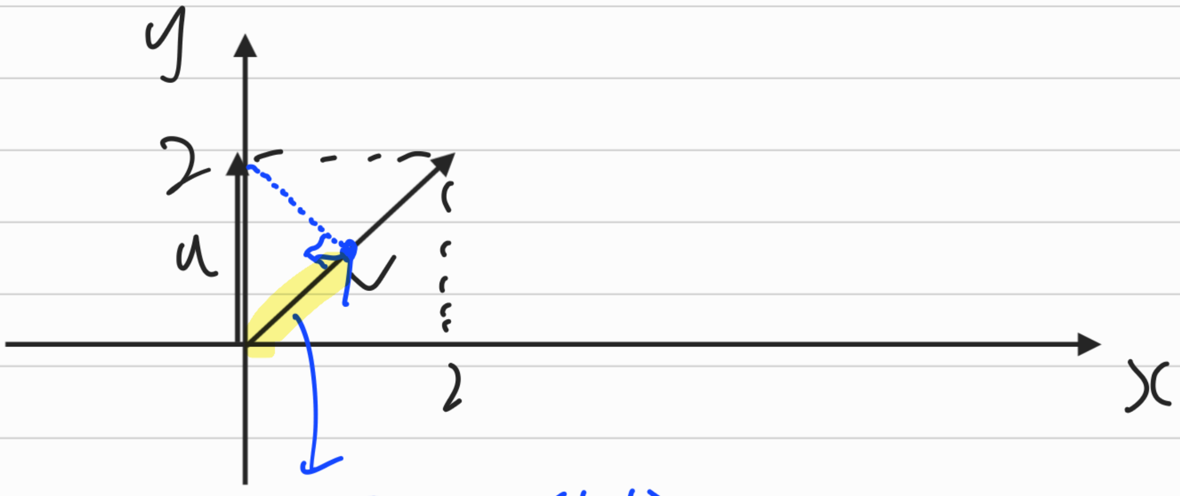
ex) $u = \langle 0, 2\rangle$, $v = \langle 2, 2\rangle$.

$\mathrm{proj}_v u$? (u의 v로의 사영).

$$\mathrm{proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right)v$$

* $u \cdot v = \langle 0, 2\rangle \cdot \langle 2, 2\rangle = 0 + 4 = 4$.
* $|v| = 2\sqrt{2} \Rightarrow |v|^2 = 8$.

$$\therefore \mathrm{proj}_v u = \frac{4}{8}v = \frac{1}{2}v = \langle 1, 1\rangle.$$



$proj_v u = \langle 1, 1 \rangle$.

* 내적

벡터 x 벡터 → 스칼라.

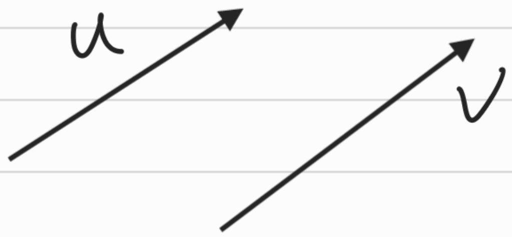
* 외적

벡터 x 벡터 → 벡터.

벡터의 외적

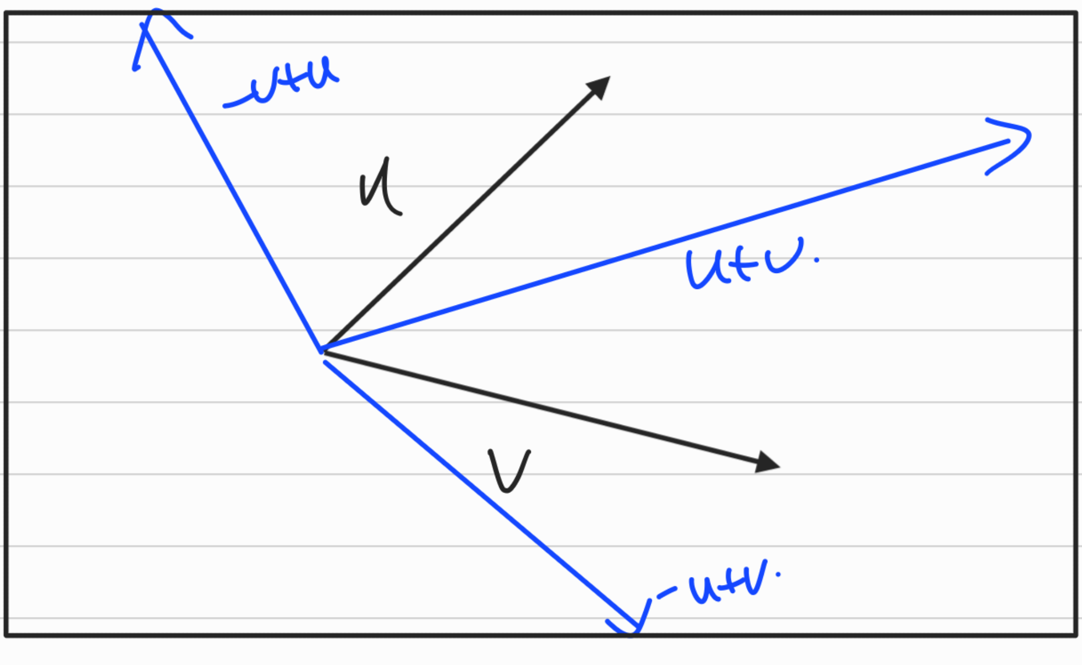
1. 벡터의 외적

u, v = 벡터, $u \neq 0$ $u = kv$ (u & v 평행)

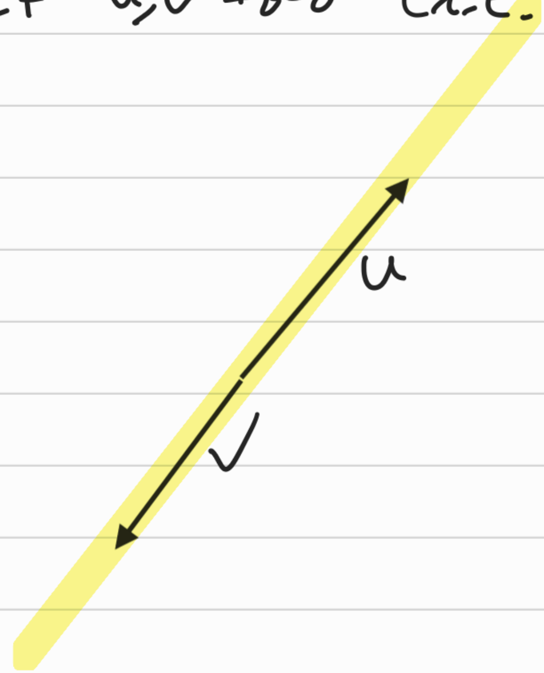


u, v = 평행 X ⇒ u & v : 한 평면 결정.

$\{k_1 u + k_2 v \mid k_1, k_2 \in \mathbb{R}\}$.

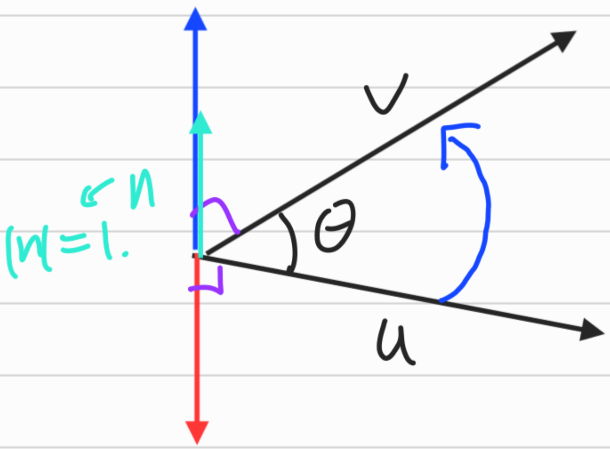


If u, v 평행 (i.e. $\exists k \in \mathbb{R}$ s.t. $u = kv$.



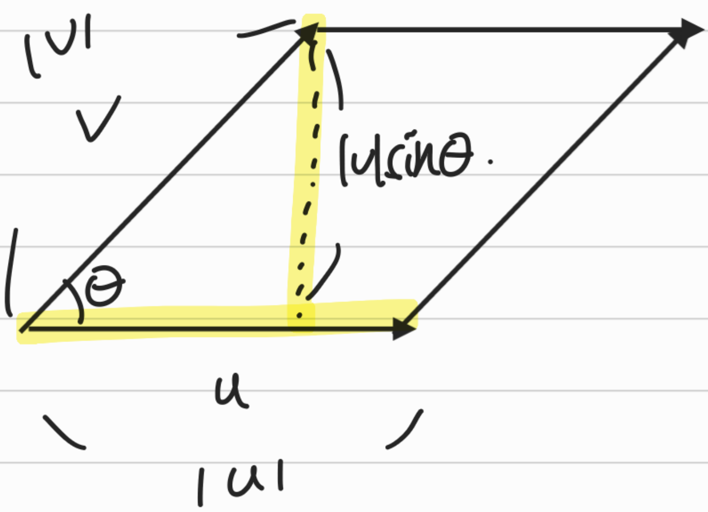
$\{au+bv \mid a, b \in \mathbb{R}\}$ = 직선.

* 오른손 법칙.



* 외적

$u \times v$: 벡터. [크기 : $|u||v|\sin\theta$. (u, v가 이루는 평행사변형 넓이)
방향 : n의 방향. (오른손법칙).



2. 벡터의 평행.

$u, v \neq 0$.

$$u \times v = 0 \Rightarrow u \,/\!/\, v.$$

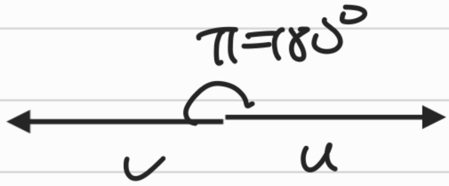
벡터. 평행.

pf) $u \times v = 0$

$\Rightarrow |u \times v| = |0| = 0.$

$\Rightarrow |u \times v| = \underbrace{|u||v|}_{\text{양}}\sin\theta = 0.$ $\quad \to 0, \pi.$

$\Rightarrow \sin\theta = 0. \quad \therefore \theta = 0 \text{ or } \pi.$

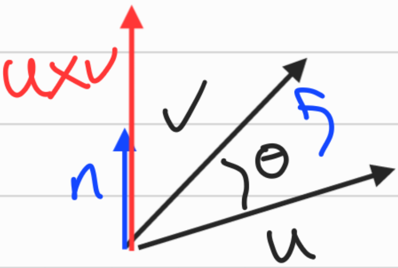


3. 외적의 성질.

u, v, w : 벡터, r&s : 스칼라.

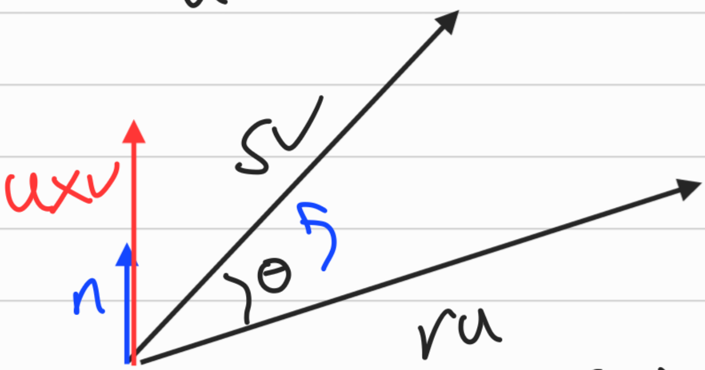
1) $(ru) \times (sv) = (rs)(u \times v).$

pf) $(r, s > 0).$



$|u \times v| = |u||v|\sin\theta$

$(rs)(u \times v)$ $\begin{cases} \text{크기} = |rs||u||v|\sin\theta. \\ \text{방향} = n \end{cases}$



$|(ru) \times (sv)|$

$= |ru||sv|\sin\theta.$

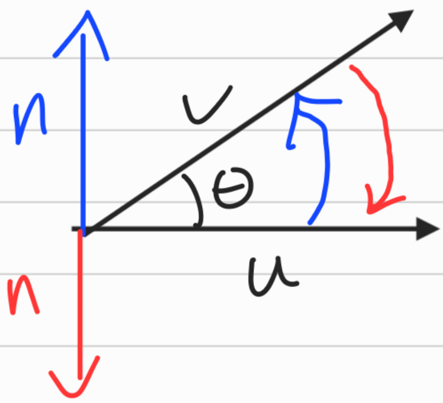
$= |r||u||s||v|\sin\theta.$

$(ru) \times (sv)$ $\begin{cases} \text{크기} : |rs||u||v|\sin\theta \\ \text{방향} : n. \end{cases}$

2) $u \times (v+w) = u\times v + u \times w$ (분배법칙).

3) $v \times u = -(u \times v)$.

pf)



$|v \times u| = |v||u| \sin\theta$

$|u \times v| = |u||v| \sin\theta$.

4) $(v+w)\times u = v\times u + w \times u$.

pf) $(v+w)\times u = -(u\times(v+w))$

$= -(u\times v + u \times w)$

$= -(u\times v) - (u \times w)$

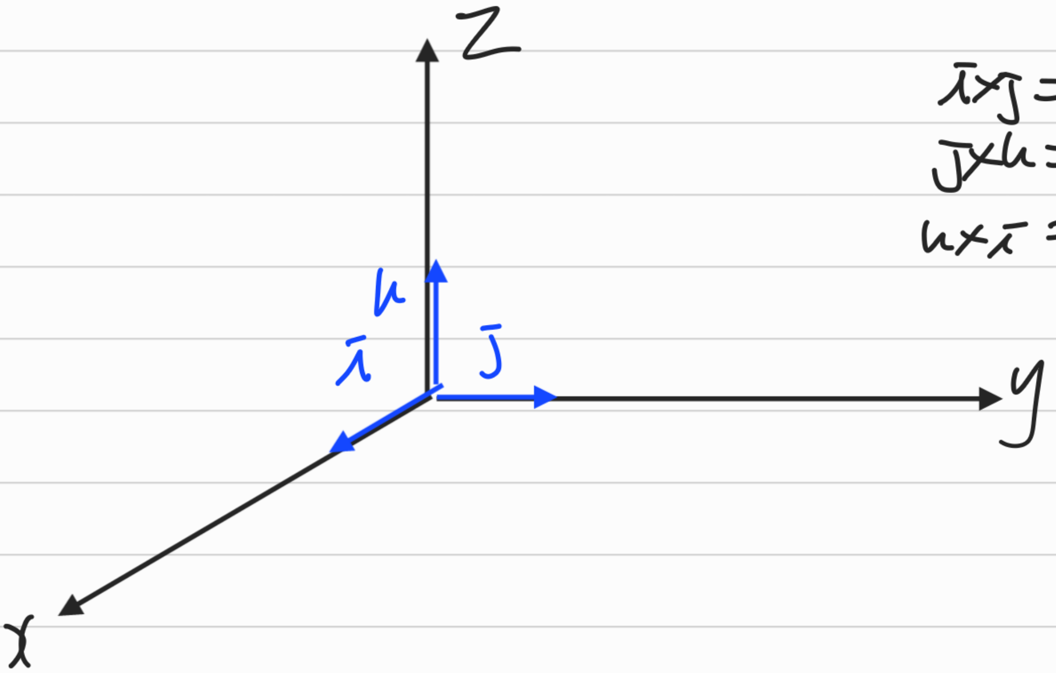
$= v\times u + w \times u$.

5) $0 \times u = 0$.

pf) $|0\times u| = |0||u|\sin\theta$

$= 0$.

6) $u\times(v\times w) = (u\cdot w)v - (u\cdot v)w$ (증명생략).



$i \times j = k$

$j \times k = i$

$k \times i = j$ (check!)

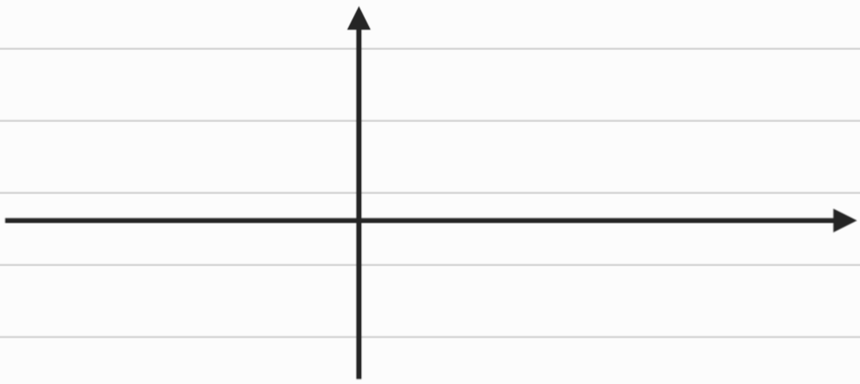
[크기
  방향

n차원 유클리드 공간 $\mathbb{R}^n$

$\mathbb{R}$:

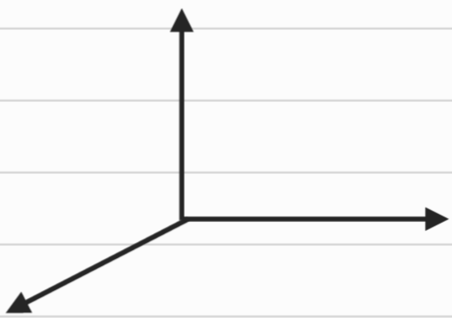
(1)

$\mathbb{R}^2$:



(1,2)

$\mathbb{R}^3$



(1,2,3)

⋮

$\mathbb{R}^n$?

1. n차원 벡터.

$v = (\underbrace{v_1, v_2, \dots, v_n}_{n\text{-tuple}})$, $v_i \in \mathbb{R}$ $\forall i$.

2. n차원 벡터의 연산법칙 (ok!)

3. 선형결합 (ok!)

계수.

$w = \underbrace{c_1 v_1 + c_2 v_2 + \dots + c_r v_r}_{\hookrightarrow\ v_1, \dots, v_r\text{의 선형결합.}}$. ($v_1, \dots, v_r$ = n차원벡터.
c_i = 스칼라.

4. n차원벡터의 크기

1) 크기.

$v = (v_1, v_2, \ldots, v_n)$

$|v| = \|v\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}.$

v의 norm, magnitude, length.

ex) $v = (1, 0, -2, -2)$의 크기.

$\|v\| = \sqrt{1^2 + 0^2 + (-2)^2 + (-2)^2}$

$= \sqrt{9} = 3.$

2) 크기의 성질.

$v \in \mathbb{R}^n, \; k \in \mathbb{R}.$

(a) $\|v\| \geq 0. \quad \forall v \in \mathbb{R}^n.$

(b) $\|v\| = 0$ iff $v = 0.$ (check!)

if and only if.

(c) $\|kv\| = |k| \, \|v\|$

3) 표준 단위벡터.

$e_1 = (1, 0, \ldots, 0) \in \mathbb{R}^n.$

$e_2 = (0, 1, 0, \ldots, 0) \in \mathbb{R}^n.$

$\vdots$

$e_n = (0, 0, \ldots, 0, 1) \in \mathbb{R}^n.$

$\forall v = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n.$

$v = v_1 e_1 + v_2 e_2 + \cdots + v_n e_n.$

$e_1, \ldots, e_n$의 선형결합.

5. n차원벡터의 거리.

$u = (u_1, u_2, \ldots, u_n), \quad v = (v_1, v_2, \ldots, v_n)$

$d(u, v) = \|u - v\| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots + (u_n - v_n)^2}.$

ex) $u = (1, 0, 0, 1), \; v = (2, 1, 0, 3).$

$d(u, v) = \sqrt{(1-2)^2 + (0-1)^2 + (0-0)^2 + (1-3)^2}$

$= \sqrt{1^2 + 1^2 + 0^2 + 2^2}$

$= \sqrt{6}.$

6. n차원벡터의 내적.

$u = (u_1, \dots, u_n)$, $v = (v_1, \dots, v_n)$.

$\Rightarrow u \cdot v = u_1 v_1 + \cdots + u_n v_n$.

ex) $u = (1, 0, 0, 1)$, $v = (0, 1, 1, 0)$

$u \cdot v = 1 \times 0 + 0 \times 1 + 0 \times 1 + 1 \times 0$

$= 0$.

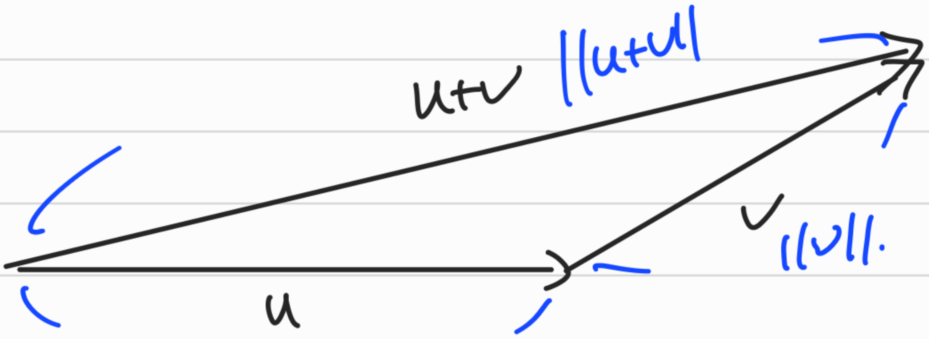
$u \perp v$.

7. 삼각부등식.

$u, v, w \in \mathbb{R}^n$.

(a) $\|u+v\| \le \|u\| + \|v\|$.

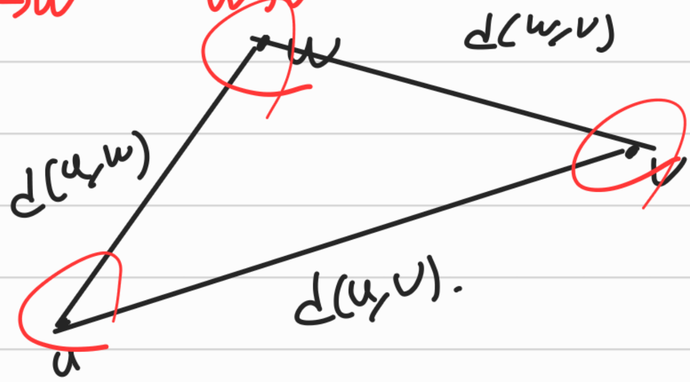
pf)



정확X. ||u|| 정위치 O

(b) $\underset{u \to v}{d(u,v)} \le \underset{u \to w}{d(u,w)} + \underset{w \to v}{d(w,v)}$.

pf)



벡터 [크기
방향. (시점, 종점) Component Form.

내적 외적.