In Exercises 9–16, find the component form of the vector.

(origin: 원점)

(midpoint: 중점)

(segment: 선분)

(sum: 합)

9. The vector $\overrightarrow{PQ}$, where $P = (1, 3)$ and $Q = (2, -1)$

10. The vector $\overrightarrow{OP}$ where O is the origin and P is the midpoint of segment RS, where $R = (2, -1)$ and $S = (-4, 3)$

11. The vector from the point $A = (2, 3)$ to the origin

12. The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$, where $A = (1, -1)$, $B = (2, 0)$, $C = (-1, 3)$, and $D = (-2, 2)$

(magnitude: 크기)

Vectors in the Plane

In Exercises 1–8, let $\mathbf{u} = \langle 3, -2 \rangle$ and $\mathbf{v} = \langle -2, 5 \rangle$. Find the **(a)** component form and **(b)** magnitude (length) of the vector.

1. $3\mathbf{u}$

2. $-2\mathbf{v}$

3. $\mathbf{u} + \mathbf{v}$

4. $\mathbf{u} - \mathbf{v}$

5. $2\mathbf{u} - 3\mathbf{v}$

6. $-2\mathbf{u} + 5\mathbf{v}$

7. $\frac{3}{5}\mathbf{u} + \frac{4}{5}\mathbf{v}$

8. $-\frac{5}{13}\mathbf{u} + \frac{12}{13}\mathbf{v}$

Vectors in Space

In Exercises 17–22, express each vector in the form $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

17. $\overrightarrow{P_1P_2}$ if P_1 is the point $(5, 7, -1)$ and P_2 is the point $(2, 9, -2)$

18. $\overrightarrow{P_1P_2}$ if P_1 is the point $(1, 2, 0)$ and P_2 is the point $(-3, 0, 5)$

19. $\overrightarrow{AB}$ if A is the point $(-7, -8, 1)$ and B is the point $(-10, 8, 1)$

20. $\overrightarrow{AB}$ if A is the point $(1, 0, 3)$ and B is the point $(-1, 4, 5)$

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연습문제 9-16:

벡터의 Component Form을 구하십시오.

연습문제 1-8:

주어진 상황에서 벡터의 (a) Component Form와 (b) 크기(길이)를 찾으십시오.

Thomas' Calculus - 12.3 The Dot Product

Dot Product and Projections

In Exercises 1–8, find

a. $\mathbf{v} \cdot \mathbf{u}$, $|\mathbf{v}|$, $|\mathbf{u}|$

b. the cosine of the angle between $\mathbf{v}$ and $\mathbf{u}$

c. the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$

d. the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

1. $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} + \sqrt{5}\mathbf{k}, \quad \mathbf{u} = -2\mathbf{i} + 4\mathbf{j} - \sqrt{5}\mathbf{k}$

2. $\mathbf{v} = (3/5)\mathbf{i} + (4/5)\mathbf{k}, \quad \mathbf{u} = 5\mathbf{i} + 12\mathbf{j}$

3. $\mathbf{v} = 10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k}, \quad \mathbf{u} = 3\mathbf{j} + 4\mathbf{k}$

4. $\mathbf{v} = 2\mathbf{i} + 10\mathbf{j} - 11\mathbf{k}, \quad \mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

5. $\mathbf{v} = 5\mathbf{j} - 3\mathbf{k}, \quad \mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

6. $\mathbf{v} = -\mathbf{i} + \mathbf{j}, \quad \mathbf{u} = \sqrt{2}\mathbf{i} + \sqrt{3}\mathbf{j} + 2\mathbf{k}$

7. $\mathbf{v} = 5\mathbf{i} + \mathbf{j}, \quad \mathbf{u} = 2\mathbf{i} + \sqrt{17}\mathbf{j}$

8. $\mathbf{v} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \right\rangle, \quad \mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} \right\rangle$

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연습문제 1-8:

(b) v, u가 이루는 각의 코사인 값

(c) v 방향의 u의 스칼라 컴포넌트 (proj v u의 크기)

11. Let $\mathbf{u} = (-3, 2, 1, 0)$, $\mathbf{v} = (4, 7, -3, 2)$, and $\mathbf{w} = (5, -2, 8, 1)$. Find the components of

(a) $\mathbf{v} - \mathbf{w}$

(b) $-\mathbf{u} + (\mathbf{v} - 4\mathbf{w})$

(c) $6(\mathbf{u} - 3\mathbf{v})$

(d) $(6\mathbf{v} - \mathbf{w}) - (4\mathbf{u} + \mathbf{v})$

17. Let $\mathbf{u} = (1, -1, 3, 5)$ and $\mathbf{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\mathbf{u} + b\mathbf{v} = (1, -4, 9, 18)$.

21. Show that there do not exist scalars c_1, c_2, and c_3 such that

$$c_1(-2, 9, 6) + c_2(-3, 2, 1) + c_3(1, 7, 5) = (0, 5, 4)$$

True-False Exercises

TF. In parts (a)–(k) determine whether the statement is true or false, and justify your answer.

(d) The vectors $\mathbf{v} + (\mathbf{u} + \mathbf{w})$ and $(\mathbf{w} + \mathbf{v}) + \mathbf{u}$ are the same.

(e) If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

(h) If $(a, b, c) + (x, y, z) = (x, y, z)$, then (a, b, c) must be the zero vector.

(i) If k and m are scalars and $\mathbf{u}$ and $\mathbf{v}$ are vectors, then

$$(k + m)(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + m\mathbf{v}$$

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11. u = (−3, 2, 1, 0), v = (4, 7, −3, 2), w = (5, −2, 8, 1) 일 때,

(a) v - w의 성분을 구하시오.

(b) -u + (v - 4w)의 성분을 구하시오.

(c) 6(u - 3v)의 성분을 구하시오.

(d) (6v - w) - (4u + v)의 성분을 구하시오.

17. u = (1, −1, 3, 5), v = (2, 1, 0, −3)일 때, 스칼라 a와 b를 찾아서 au + bv = (1, −4, 9, 18)을 만족하도록 하시오.

21. c1(−2, 9, 6) + c2(−3, 2, 1) + c3(1, 7, 5) = (0, 5, 4)를 만족하는 스칼라 c1, c2, c3가 존재하지 않음을 보이시오.

True-False Questions

(d) 벡터 v + (u + w)와 (w + v) + u가 같음을 보이시오.

(e) 만약 u + v = u + w이면, v = w임을 보이시오.

(h) 만약 (a, b, c) + (x, y, z) = (x, y, z)이면, (a, b, c)는 영벡터여야 함을 보이시오.

(i) 만약 k와 m이 스칼라이고, u와 v가 벡터이면, (k + m)(u + v) = ku + mv임을 보이시오.

▶ In Exercises 3–4, evaluate the given expression with $\mathbf{u} = (2, -2, 3)$, $\mathbf{v} = (1, -3, 4)$, and $\mathbf{w} = (3, 6, -4)$. ◀

3. (a) $\|\mathbf{u} + \mathbf{v}\|$ (b) $\|\mathbf{u}\| + \|\mathbf{v}\|$

(c) $\|-2\mathbf{u} + 2\mathbf{v}\|$ (d) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$

▶ In Exercises 5–6, evaluate the given expression with $\mathbf{u} = (-2, -1, 4, 5)$, $\mathbf{v} = (3, 1, -5, 7)$, and $\mathbf{w} = (-6, 2, 1, 1)$. ◀

5. (a) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$ (b) $\|3\mathbf{u}\| - 5\|\mathbf{v}\| + \|\mathbf{w}\|$

(c) $\|-\|\mathbf{u}\|\mathbf{v}\|$

▶ In Exercises 9–10, find $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \cdot \mathbf{u}$, and $\mathbf{v} \cdot \mathbf{v}$. ◀

9. (a) $\mathbf{u} = (3, 1, 4)$, $\mathbf{v} = (2, 2, -4)$

(b) $\mathbf{u} = (1, 1, 4, 6)$, $\mathbf{v} = (2, -2, 3, -2)$

▶ In Exercises 15–16, determine whether the expression makes sense mathematically. If not, explain why. ◀

15. (a) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$ (b) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$

(c) $\|\mathbf{u} \cdot \mathbf{v}\|$ (d) $(\mathbf{u} \cdot \mathbf{v}) - \|\mathbf{u}\|$

True-False Exercises

TF. In parts (a)–(j) determine whether the statement is true or false, and justify your answer.

(norm: 크기)

(a) If each component of a vector in R^3 is doubled, the norm of that vector is doubled.

(c) Every vector in R^n has a positive norm.

(f) The expressions $(\mathbf{u} \cdot \mathbf{v}) + \mathbf{w}$ and $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ are both meaningful and equal to each other.

(g) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

(h) If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

(j) For all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in R^n, we have

$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\| + \|\mathbf{w}\|$$

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(a) R3의 벡터의 모든 성분을 두 배로 곱하면, 그 벡터의 노름(norm)도 두 배가 된다.

(c) Rn의 모든 벡터는 양의 노름을 가진다.

(f) 식 (u · v) + w와 u · (v + w)는 모두 의미가 있으며 서로 같다.

(g) 만약 u · v = u · w이면, v = w이다.

(h) 만약 u · v = 0이면, u = 0이거나 v = 0이다.

(j) Rn의 모든 벡터 u, v, w에 대해 다음이 성립한다.

(orthogonal: 수직인, 직교하는)

▶ In Exercises 1–2, determine whether **u** and **v** are orthogonal vectors. ◀

1. (a) $\mathbf{u} = (6, 1, 4),\ \mathbf{v} = (2, 0, -3)$

(b) $\mathbf{u} = (0, 0, -1),\ \mathbf{v} = (1, 1, 1)$

(c) $\mathbf{u} = (3, -2, 1, 3),\ \mathbf{v} = (-4, 1, -3, 7)$

(d) $\mathbf{u} = (5, -4, 0, 3),\ \mathbf{v} = (-4, 1, -3, 7)$

▶ In Exercises 13–14, find $\|\text{proj}_{\mathbf{a}}\mathbf{u}\|$. ◀

13. (a) $\mathbf{u} = (1, -2),\ \mathbf{a} = (-4, -3)$

(b) $\mathbf{u} = (3, 0, 4),\ \mathbf{a} = (2, 3, 3)$

▶ In Exercises 15–20, find the vector component of **u** along **a** and the vector component of **u** orthogonal to **a**. ◀

15. $\mathbf{u} = (6, 2),\ \mathbf{a} = (3, -9)$

16. $\mathbf{u} = (-1, -2),\ \mathbf{a} = (-2, 3)$

17. $\mathbf{u} = (3, 1, -7),\ \mathbf{a} = (1, 0, 5)$

18. $\mathbf{u} = (2, 0, 1),\ \mathbf{a} = (1, 2, 3)$

19. $\mathbf{u} = (2, 1, 1, 2),\ \mathbf{a} = (4, -4, 2, -2)$

20. $\mathbf{u} = (5, 0, -3, 7),\ \mathbf{a} = (2, 1, -1, -1)$

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Vector component of u along a: a 방향의 u의 벡터 Component (proj a u)

Vector component of u orthogonal to a: a와 수직인 u의 벡터 Component (u - proj a u)

True-False Exercises

TF. In parts (a)–(g) determine whether the statement is true or false, and justify your answer.

(a) The vectors $(3, -1, 2)$ and $(0, 0, 0)$ are orthogonal.

(b) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors, then for all nonzero scalars k and m, $k\mathbf{u}$ and $m\mathbf{v}$ are orthogonal vectors.

(c) The orthogonal projection of $\mathbf{u}$ on $\mathbf{a}$ is perpendicular to the vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$.

(d) If $\mathbf{a}$ and $\mathbf{b}$ are orthogonal vectors, then for every nonzero vector $\mathbf{u}$, we have

$$\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u})) = \mathbf{0}$$

(e) If $\mathbf{a}$ and $\mathbf{u}$ are nonzero vectors, then

$$\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}(\mathbf{u})) = \text{proj}_{\mathbf{a}}(\mathbf{u})$$

(f) If the relationship

$$\text{proj}_{\mathbf{a}}\mathbf{u} = \text{proj}_{\mathbf{a}}\mathbf{v}$$

holds for some nonzero vector $\mathbf{a}$, then $\mathbf{u} = \mathbf{v}$.

(g) For all vectors $\mathbf{u}$ and $\mathbf{v}$, it is true that

$$\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$$

한글번역)

(a) 벡터 (3, -1, 2)와 (0, 0, 0)은 직교한다.

(b) 만약 u와 v가 직교하는 벡터이면, 모든 0이 아닌 스칼라 k와 m에 대해, ku와 mv는 직교하는 벡터이다.

(c) u의 a에 대한 직교 성분과 그에 수직인 a에 대한 정사영(projection)은 서로 직교한다.

(d) 만약 a와 b가 직교하는 벡터이면, 모든 0이 아닌 벡터 u에 대해, projb(u)에 대한 a에 대한 정사영(projection)은 0이다.

(e) 만약 a와 u가 0이 아닌 벡터이면, proja(u)에 대한 a에 대한 정사영은 proja(u)이다.

(f) 만약 어떤 0이 아닌 벡터 a에 대해 projau = projav가 성립하면, u = v이다.

(g) 모든 벡터 u와 v에 대해, 다음이 성립한다.

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