[Mathematical Economics]

01. Find the following systems of difference equations.

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.2 \\ 0.1 & 0.7 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ with } \begin{pmatrix} x_o \\ y_o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

1.Find the eigenvalues and eigenvectors of coefficient matrix. **(10 points)**

2.Solve the difference equation for x_t, y_t . **(20 points)**

02. For Matrix B given below, Calculate the determinant of B^{-1}. **(20 points)**

$$B = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 1 & 0 & 3 & 1 \\ -1 & 3 & 2 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

03. Find the taylor series for $y = \log_2 x$ at $x = 1$. **(10 points)**

04. Suppose that we minimize the strictly convex function $F(x,y,z)$ subject to $x+y+pz=b$ in this problem. p,b are all positive constants. x,y,z are positive. $F(x,y,z)$ satisfies $\frac{\partial F}{\partial x} > 0$, $\frac{\partial F}{\partial y} > 0$, $\frac{\partial F}{\partial z} > 0$. Let $V(p,b)$ the minimized value of $F(x,y,z)$ given $g(p,b)$. Find the signs of $\frac{\partial V}{\partial p}$ and $\frac{\partial V}{\partial b}$. **(10 points)**

05.Find the following optimization problem. **(20 points)**

$$Maximize\ x^a + y^a,\ (\forall\, 0 < a < 1, constant)$$
$$s.t\ \ x + y \leq 1,\ x \geq 0,\ y \geq 0$$

1.Write Lagrangian and First-order conditions. **(10 points)**

2.Find (x,y) that satisfies all of first-order conditions. **(15 points)**

3.Check second-order-conditions and obtain the solution of (x^*,y^*). **(15 points)**

[Statistics]

01. Let X discrete Random Variable where probability mass function(**p,m,f**) is

$$P(X=0)=\lambda+(1-\lambda)e^{-\theta}$$

$$P(X=x)=\frac{(1-\lambda)e^{-\theta}\theta^{x}}{x!}I(x=1,2,.....)$$

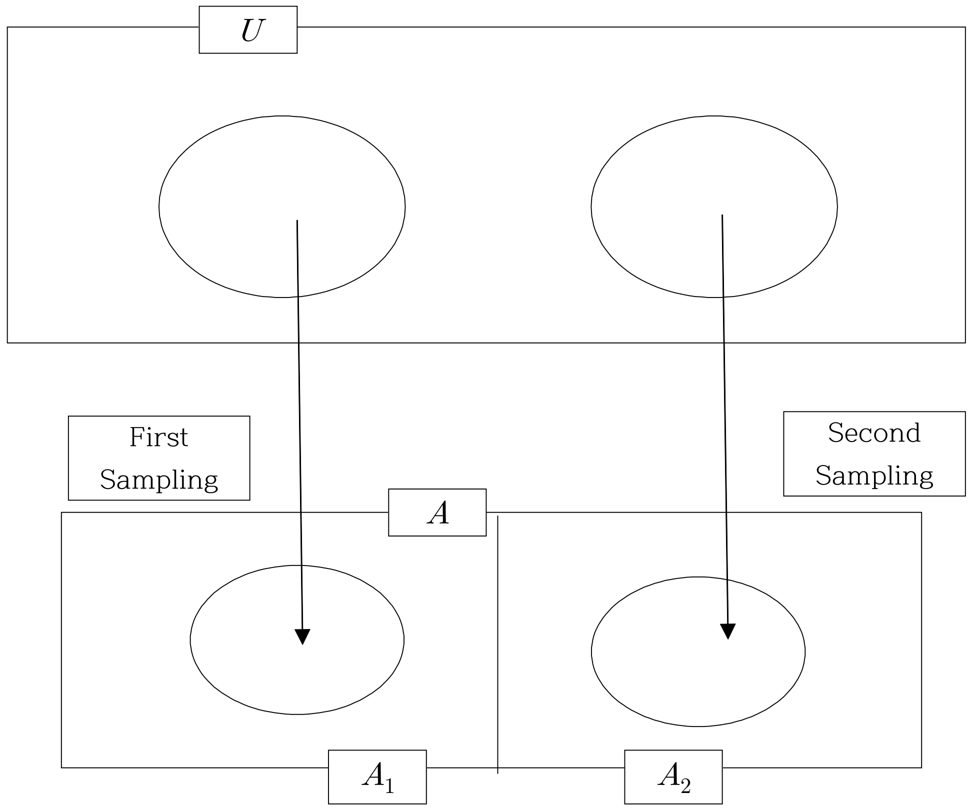
with $0\le\lambda<1,\ \theta>0.$

1.What is the Expectation and Variance, $E(X),\ Var(X)$?

2.What is the probability function of X given, $X>0$, denoted by $P(X|X>0)$?

3.Now assume that $\lambda=0$. obtain the conditional Expectation and the conditional variance denoted by $E(X|X>0),\ Var(X|X>0)$.

02. Let U be a finite population of size N, we define the following sampling design. we first select a sample A_1 according to simple Random sampling without replacement of fixed size n_1. we then select a sample A_2 in U outside of A_1 according to simple Random sampling without replacement of fixed size n_2. The Final sample A consists of A_1 and A_2 with size of $n(=n_1+n_2)<N$.



1. What is the sampling distribution of A_1, denoted by $P(A_1)$?

2. What is the sampling distribution of A_2 given A_1, denoted by $P(A_2 | A_1)$?

3. What is the sampling distribution of A?

4.Suppose that we now directly draw a sample B according to a simple Random Sampling without of fixed size $n(=n_1+n_2)<N$. obtain the sampling distribution of B and compare it with the sampling distribution of A.

5.We define the estimator of the finite population mean of y, $\overline{y}$ by

$$\overline{y_\lambda}=\lambda\overline{y_1}+(1-\lambda)\overline{y_2}$$

with $0<\lambda<1$. where $\overline{y_1}$ is the sample mean of y in A_1. $\overline{y_2}$ is the sample mean of y in A_2. show that $\overline{y_\lambda}$ is the unbiased estimator for $\overline{y}$ for any λ.

6.Express variance of $V(\overline{y_\lambda})$ in terms of $V(\overline{y_1}), V(\overline{y_2}), Cov(\overline{y_1},\overline{y_2})$. Note that you have to obtain the closed forms of $Var(\overline{y_1}), Var(\overline{y_2}), Cov(\overline{y_1},\overline{y_2})$

7.Find the optimal value of λ in terms of n_1,n_2 that satisfies minimizing variance of $\overline{y_\lambda}$, $V(\overline{y_\lambda})$.