< 통계학 >

1. [30 점] 간단한 베이즈 정리 문제

2. [70 점]

Suppose we collected data on midterm and final exam scores for $n = 34$ students. A simple linear regression model that predicts the final score (F) from the midterm score (M) is given by

$$F_i = \beta_0 + \beta_1 M_i + \varepsilon_i \qquad \varepsilon_i \sim N(0, \sigma^2)$$

and a simple linear regression model that predicts the midterm score from the final score is given by

$$M_i = \gamma_0 + \gamma_1 F_i + \epsilon_i \qquad \epsilon_i \sim N(0, \tau^2)$$

where M_i denotes the midterm exam scores of the i^{th} student. F_i denotes the final exam score of the i^{th} student. The midterm and final exam score is positively correlated.

a. Below is a portion of outputs for a simple linear regression analysis to predict the final score from the midterm score. Complete the following ANOVA table and discuss the results of the F-test, showing details of calculations.

Source of var	Sum of squares	df	Mean square	F-stat	P-val
Regression	1800	(2)	(5)	(7)	(8)
Residual	3200	(3)	(6)		
Total	(1)	(4)			

b. Sample statistics for the midterm and final scores are as follows.

	Mean	Standard Deviation
Final	50	12
Mid	60	8

Using the ANOVA table in part (a) and the sample statistics above, complete the following tables, showing details of calculations. Hint : $\sqrt{42.24} = 6.5$

	Coefficient	Standard Error	t-stat	p-val	Regressing F on M
Constant	(1)	13	(3)	(5)	
M_i	(2)	0.2	(4)	(6)	

$R^2 =$ (7) Estimate of $\sigma =$ (8)

	Coefficient	Standard Error	t-stat	p-val	Regressing F on M
Constant	(9)	5	(11)	(13)	
F_i	(10)	0.1	(12)	(14)	

$R^2 =$ (15) Estimate of $\tau =$ (16)

c. Show that two estimated regression lines in part (b) are necessarily different, by using the sum of squares in the Anova table. Explain the intuition.

d. the data & results show that students in general do worse of F than M. A question of interest is whether the lower scores on the final exam is of a simple additive factor, that is, whether the difference, M-F, is uncorrelated with M. Using the results in part (b), perform an appropriate hypothesis test to see if there is any evidence to discredit this simple additive model at the significance level of 0.05. Be explicit about the $H_0 \& H_1$ you are testing.

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